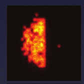
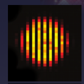
Outline

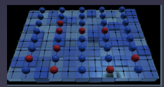
Introduction

1. **Many-Body Localisation Phase Transition in 2D**
   - Probing MBL transition using domain wall dynamics and CDW dynamics
   - Wavelength Dependence of Localization





2. **2D MBL with Coupling to a Finite Bath**
   - CDW Dynamics in the presence of a finite bath



3. **Probing Relaxation/Transport Dynamics close to MBL**

Many-Body Localisation using Ultracold Atoms in Quasiperiodic Potentials

M. Schreiber et al. Science **349**, 842 (2015)
P. Bordia et al. Phys. Rev. Lett. **116**, 140401 (2016)

E. Altman

R. Vosk

M. Fischer

Motivation

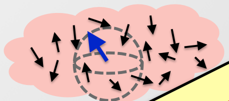
Thermalization

Quantum correlations in local d.o.f are rapidly lost as these get entangled with the rest of the system.

↓

Classical hydro description of remaining slow modes (conserved quantities, and order parameters).

Many-body localization



Local quantum in... persists ind...

Statistical Physics Fails!

Ne... **quantum** description of the long time dynamics!

?

The many-body localization transition = elusive interface between quantum and classical worlds

Gornyi et al. Phys. Rev. Lett. 2005
Basko et al. Ann. of Physics 2006
Nandkishore et al., Annu. Rev. Cond. Mat. 2015
Altman et al. Annu. Rev. Cond. Mat. 2015

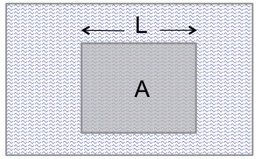
LMU

Eigenstate Thermalisation Hypothesis

Deutsch (91), Srednicki (94,98), Rigol, Dunjko & Olshanii (2009), D'Alessio, Kafri, Polkovnikov, Rigol, Adv. Phys. **65**, 239 (2016)

$$\rho_A = \frac{1}{Z_A} e^{-\beta H_A}$$

$$S_A \equiv \mathrm{tr}\left[\rho_A \ln \rho_A\right] \propto L^d$$



Are there scenarios when this fails?

System fails to act as its own heat bath!

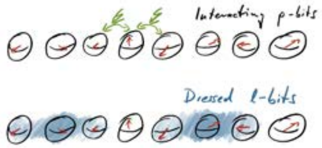
Nandkishore et al., Annu. Rev. Cond. Mat. 2015; Altman et al. Annu. Rev. Cond. Mat. 2015,

LMU

MBL — Generic Failure of Thermalization

Generic: Disorder + Interactions + High Energy Density

$$\hat{H} = \sum_i h_i \hat{S}_i^z + \sum_{i,j} J_{i,j} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j$$



$$\hat{H}_{\text{l-bit}} = \sum_i \hat{T}_i^z + \sum_{i,j} \tilde{J}_{i,j} \hat{T}_i^z \hat{T}_j^z + \sum_{n=1}^{\infty} \sum_{i,j,\{k\}} K_{i\{k\}j}^{(n)} \hat{T}_i^z \hat{T}_{k_1}^z \ldots \hat{T}_{k_n}^z \hat{T}_j^z$$

"l-bits" are **quasi-local integrals of motion**

J.Z. Imbrie, Jour. Stat. Phys. 163:998-1048 (2016)
"l-bits": Serbyn, PRL 2013 | Huse, PRB 2014 | Nandkishore, Annu. Rev. Cond. Mat. 2015

Approaching Many-Body Localization from Disordered Luttinger Liquids
C. Karrasch, J. E. Moore
Subjects: Strongly Correlated Electrons (cond-mat.str-el)
28. arXiv:1506.00592 [pdf, other]
Protection of topological order by symmetry and many-body
Andrew C. Potter, Ashvin Vishwanath
Comments: 17 pages, 4 figures
Subjects: Disordered Systems and Neural Networks (cond-m
29. arXiv:1505.07089 [pdf, other]
Dynamics of many-body localisation in a translation invariant
Merlijn van Horssen, Emanuele Levi, Juan P. Garrahan
Comments: 5 pages, 4 figures
Subjects: Statistical Mechanics (cond-mat.stat-mech); Quan
30. arXiv:1505.06343 [pdf, ps, other]
Many-body ground state localization and coexistence of loca
Yucheng Wang, Haiping Hu, Shu Chen
Comments: 5 pages, 6 figures
Subjects: Disordered Systems and Neural Networks (cond-m
31. arXiv:1505.05386 [pdf, other]
Revisiting Many-body Localization with Random Networks o
Benoît Descamps, Frank Verstraete
Comments: 3 figures
Subjects: Quantum Physics (quant-ph)
32. arXiv:1505.05147 [pdf, other]
Many-Body Localization of Symmetry Protected Topological
Kevin Slagle, Zhen Bi, Yi-Zhuang You, Cenke Xu
Comments: 5 pages 2 figures
Subjects: Strongly Correlated Electrons (cond-mat.str-el)
33. arXiv:1505.02028 [pdf, other]
Out-of-equilibrium states and quasi-many-body localization i
L. Barbiero, C. Menotti, A. Recati, L. Santos
Comments: 5 pages, 4 figures
Subjects: Quantum Gases (cond-mat.quant-gas)
34. arXiv:1504.06872 [pdf, other]
Total correlations of the diagonal ensemble herald the many-body localization transition
J. Goold, C. Gogolin, S. R. Clark, J. Eisert, A. Scardicchio, A. Silva
Subj ... ysics (quant-ph)
35. a
Many ... mobility edge
Xiao
Com
Subj ... t.quant-gas); Quantum Physics (quant-ph)
36. ar
Many body localization in the presence of a single particle mobility edge
Ranjan Modak, Subroto Mukerjee
Comments: 5 pages, 6 figures
Subjects: Disordered Systems and Neural Networks (cond-mat.dis-nn); Statistical Mechanics (cond-mat.stat-mech); Strongly Correlated Electrons (cond-mat.str-el)
37. arXiv:1503.06508 [pdf, ps, other]
Localization in a random $x-y$ model with the long-range interaction: Intermediate case between single particle and many-body problems
Alexander L. Burin
Comments: Modified version after review
Subjects: Disordered Systems and Neural Networks (cond-mat.dis-nn)
38. arXiv:1503.06147 [pdf, other]
Many-body localization characterized from a one-particle perspective

Pioneering work:
I.V. Gornyi, A. D. Mirlin, and D. G. Polyakov, PRL (2005).
D. M. Basko, I. L. Aleiner, B. L. Altschuler, Ann. Phys. (2006).

Review/intro:
D.A. Huse, R. Nandkishore, V. Oganesyan,
Annu. Rev. Cond. Mat. **6**, 15 (2015)

R. Vosk & E. Altman,
Annu. Rev. Cond. Mat. **6**, 383 (2015)

D. Abanin, E. Altman, I.B., M. Serbyn (2019)
Rev. Mod. Phys. **91**, 021001 (2019)

Experiments: Cold Atoms, Ions, NV Centers, Electronic Systems…

MBL — Interesting Questions Connected to MBL

- Nature of the phase transition (universality, diverging scales, rare regions …)
  Pal + Huse, PRB 2010 | Agarwal, PRL 2015 | Potter PRX 2015 | Vosk, PRX 2015 | Luitz, PRB 2016 …
- Entanglement dynamics in the MBL phase
  Žnidarič, PRB 2008 | Bardarson, PRL 2012 | Serbyn, PRL 2013 | Vosk, PRL 2013 | Nanduri PRB 2014 ...
- Local integrals of motion
  Serbyn, PRL 2013 | Huse, PRB 2014 | Chandran, PRB 2015 | Ros, Nucl. Phys. B 2015 …
- Stability to environmental couplings
  Nandkishore, PRB 2014 | Huse, PRB 2015 | Johri, PRL 2015 | Levi, PRL 2016 | Fischer, PRL 2016 | Luitz, PRL 2017 ...
- Coupling to small "baths"
  Nandkishore, PRB 2015 | Hyatt, PRB 2017
- Extensions of MBL to Floquet systems (time crystals, SPT phases)
  Ponte, PRL 2015 | Else, PRB 2016 | von Keyserlingk, PRB 2016 | Khemani, PRL 2016 | Yao, PRL 2017 ...

Important Points

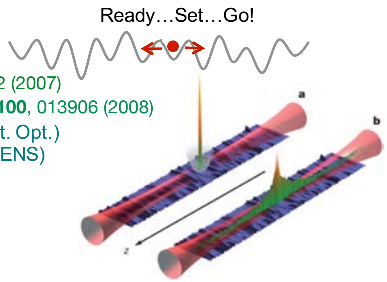
Very little theoretically known about MBL in d>1
(**stability of MBL in d>1 unclear**)

Calls for particularly precise characterization of the experiments
(validation through a quantum simulator)

Experiments (almost) isolated from environment
but **small residual coupling** limits observation time (>1000 t)

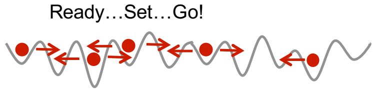
Measuring Localisation

Anderson localization:

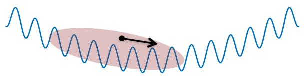


T. Schwartz et al. Nature **446**, 52 (2007)
Y. Lahini, et al. Phys. Rev. Lett. **100**, 013906 (2008)
J. Billy et. al. Nature 2008 (Inst. Opt.)
G. Roati et. al. Nature 2008 (LENS)

Many-body localization:



Fastest timescale: local probe!

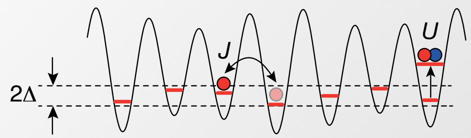


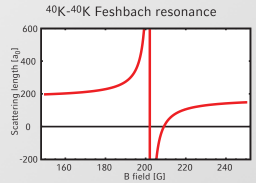
Slowest timescale: global probe

Kondov et al. (DeMarco) Phys. Rev. Lett. 114, 083002 (2015)

Our System - 1D Quasi-Disordered Fermi-Hubbard

$$H = -J\sum_{i,\sigma}\left(\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i+1,\sigma} + H.c.\right) + \Delta\sum_{i,\sigma} sin\left(2\pi\alpha i + \phi\right)\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i,\sigma} + U\sum_{i}\hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$





Without interactions U=0 : Aubry-André model

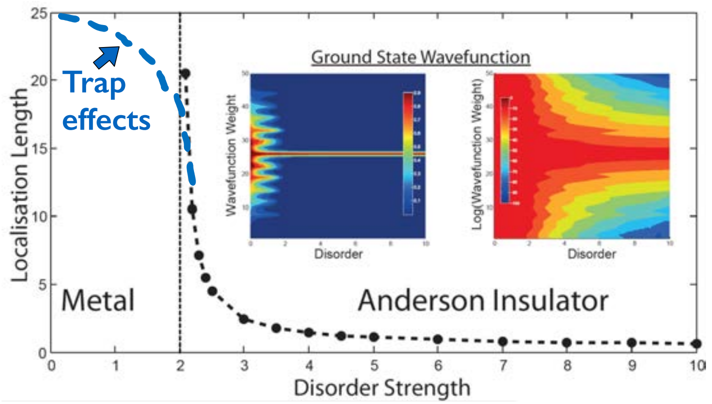
- Homogenous tunneling but quasi-random onsite energies
- α is the incommensurability ratio, irrational, in the experiment ≈ 0.721

All eigenstates extended for Δ/J < 2

All eigenstates exponentially localised for Δ/J > 2

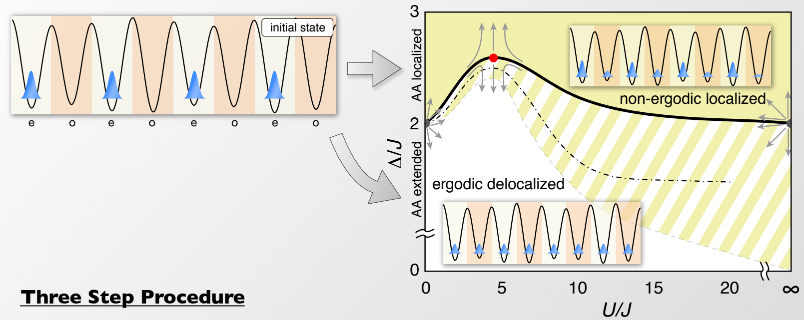
G. Roati et al. Nature (2008)

Single Particle Orbitals



$$\xi_{sp} = \ln^{-1}(\Delta/2J)$$

Probing the Interacting Aubry-André Model



Three Step Procedure

1) Prepare CDW (with different doublon densities)
2) Evolve in disorder
3) Readout CDW (disorder averaged)

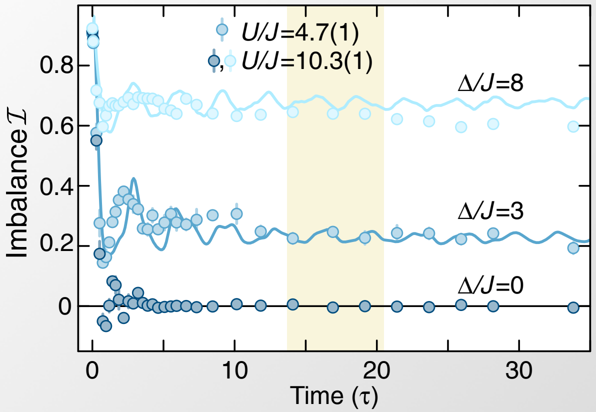
Main Observable: **Imbalance** $I = \dfrac{N_e - N_o}{N_e + N_o}$

Hamming Distance
$D(t) = \frac{N}{2}\left[1 - I(t)\right]$

(see P. Hauke & M. Heyl, PRB 2015)

MBL

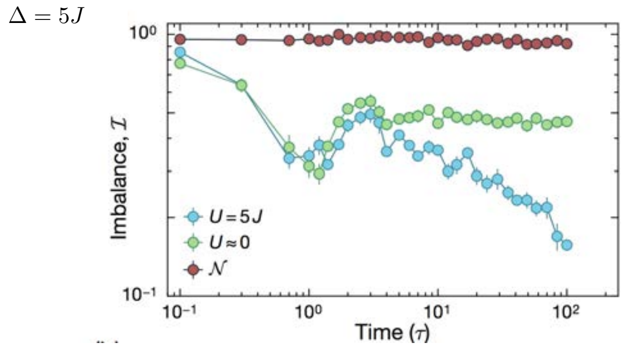
Time Evolution



Non-ergodic, non-thermalizing quantum evolution !

Relaxation

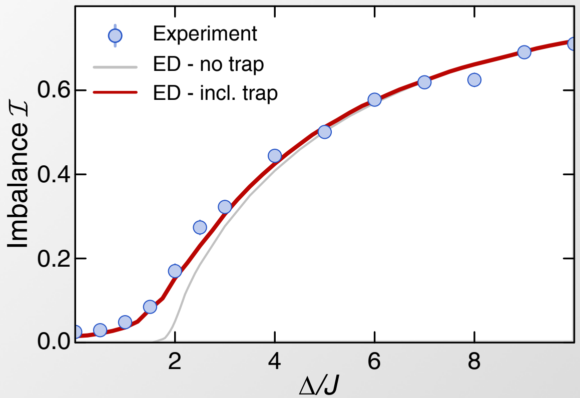
2D Quasiperiodic



Noninteracting localised - Interacting non-localised

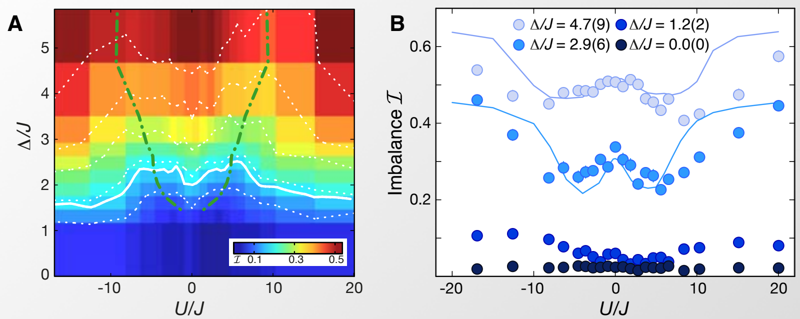
MBL

U=0 - Anderson Localization



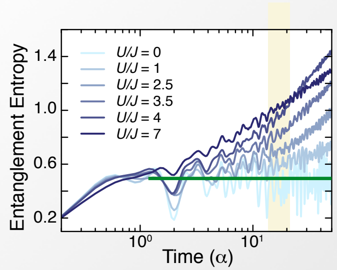
MBL

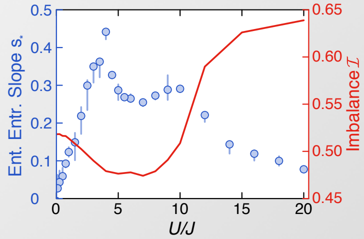
Imbalance vs U/J



1) **Localisation for all Interactions**
2) Characteristic W-shape
3) Dynamical U vs -U symmetry

Numerics - Entanglement Entropy





Characteristic log(t) entanglement entropy growth

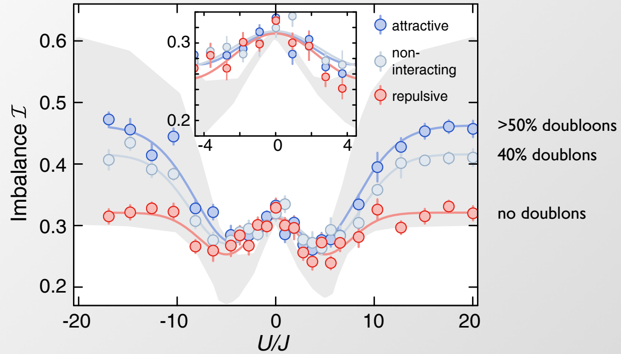
Maximum in entanglement entropy slope connected to minimum of imbalance

DMRG Simulations $\Delta/J = 5$

$$S = s_0 + s^* \log(t/\tau)$$

M Žnidarič, T Prosen, P Prelovšek, Phys. Rev. B (2008)
JH Bardarson, F Pollmann, JE Moore PRL (2012)
M Serbyn, Z Papic, DA Abanin PRL (2013)

Influence of Initial Doublon Fraction

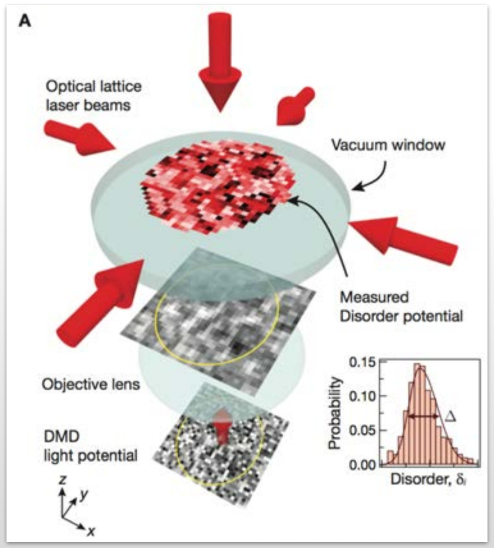


Kinetic energy of doublons for large U $J_{\mathrm{dbl}} = J^2/U$

Doublons see effectively larger disorder $\frac{J_{\mathrm{dbl}}}{\Delta} \ll \frac{J}{\Delta}$

Probing Many-Body Localisation in 2D

System Summary



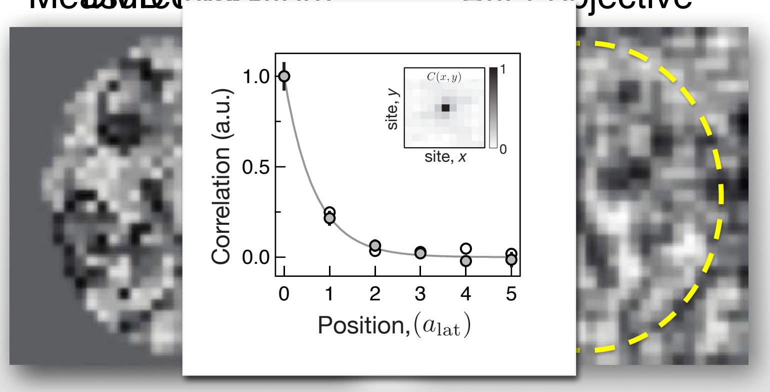
1. Prepare Domain Wall (no tunneling dynamics)
2. Turn on disorder potential
3. Lower the lattice depth (near critical point)
4. Measure atomic distribution

* Tunneling time is 6.4 ms.
* Disorder is changed for each image.
* Take 100 picture for averaging.

$$U = 24J$$
$$\Delta = 0 - 20J$$

Disorder Potential

After objective

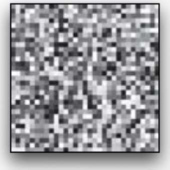


Excellent characterization of disorder !!

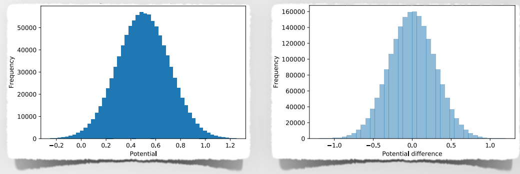
The setup

Other potentials

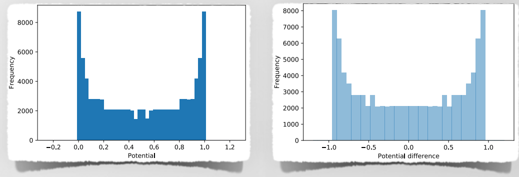
Types of potential | Potentials hist. | NN difference hist.



True random



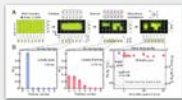
1D Quasiperiodic



MBL

Probing Non-Thermalization in AMO System

Very hard to probe whether the system has thermalised!
(but possible for small system sizes)



A. Kaufman *et al.* Science 2016

1) Start with recognisable (density) pattern

2) Evolve until steady state is reached (*always exp. limited)

3) Analyse if any remnant pattern detectable

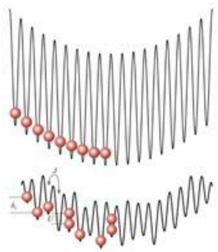
MBL

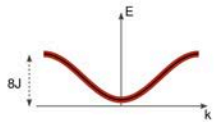
Probing Thermalization far from Equilibrium

Evolution under 2d disordered Bose-Hubbard:

$$\hat{H} = -J\sum_{<i,j>} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) + \sum_i(\delta_i + V_i)\hat{n}_i$$

System preparation far from equilibrium

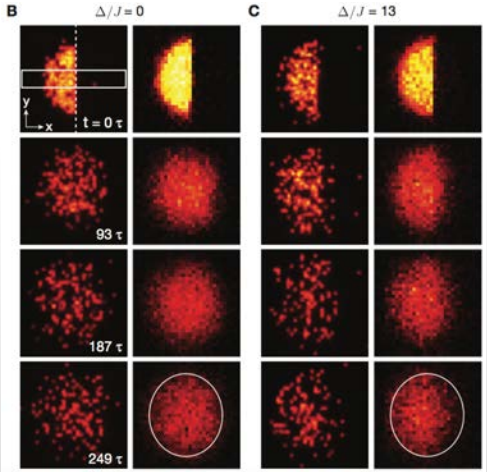




Infinite temperature wrt.
kinetic energy and disorder

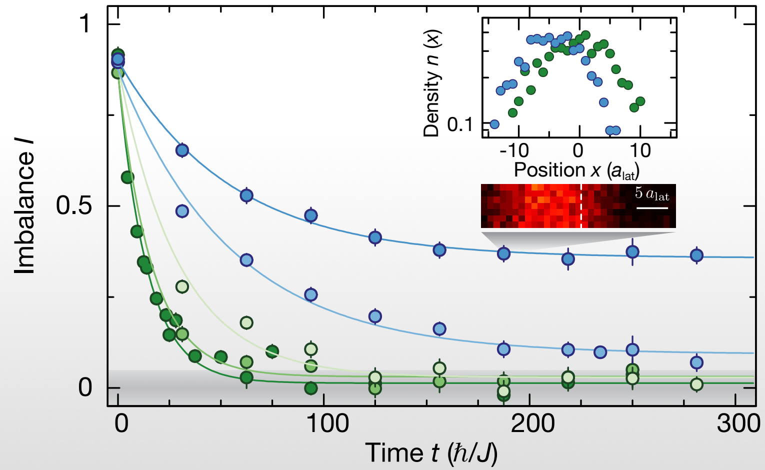
See also experiments on ground state Bose-Glass:
Kondov et al. Phys. Rev. Lett. (2015)

Domain Wall Dynamics



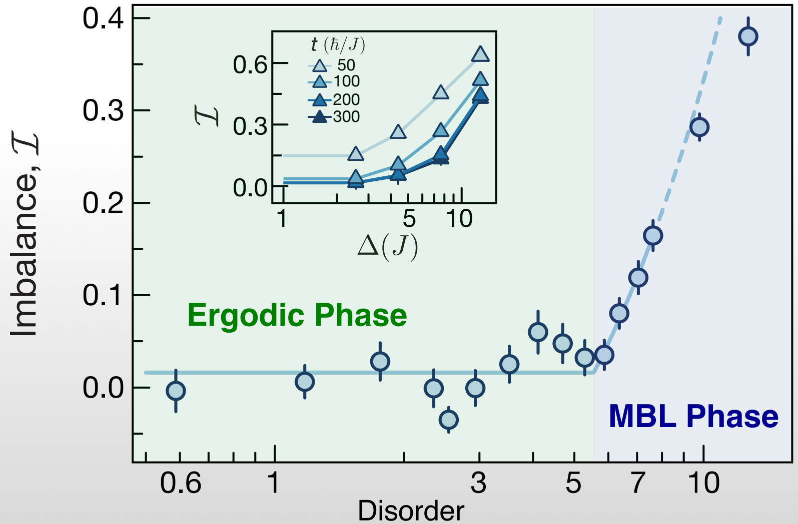
Without disorder **With disorder**

Domain Wall Imbalance

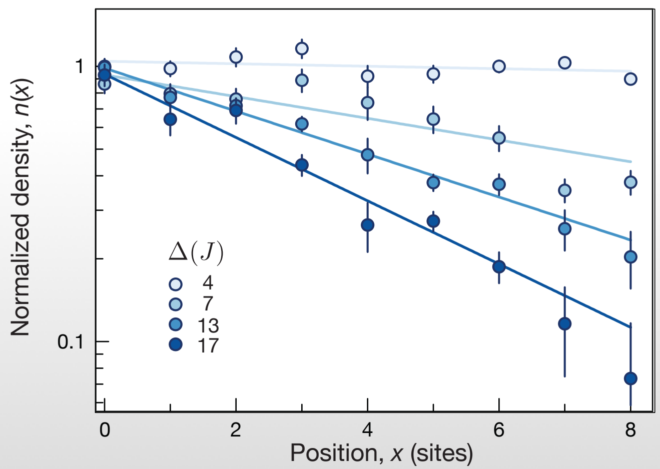


Numerics: **1d+ladder** tDMRG: Hausschild *et al.* PRB (2016); **2d** T. Wahl *et al.* arXiv:1711.02678, Yan et al PRL (2017)

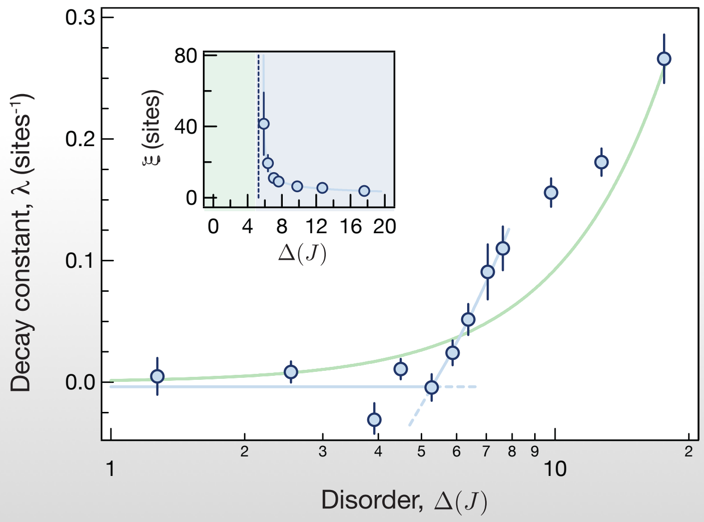
Delocalization-to-Localization



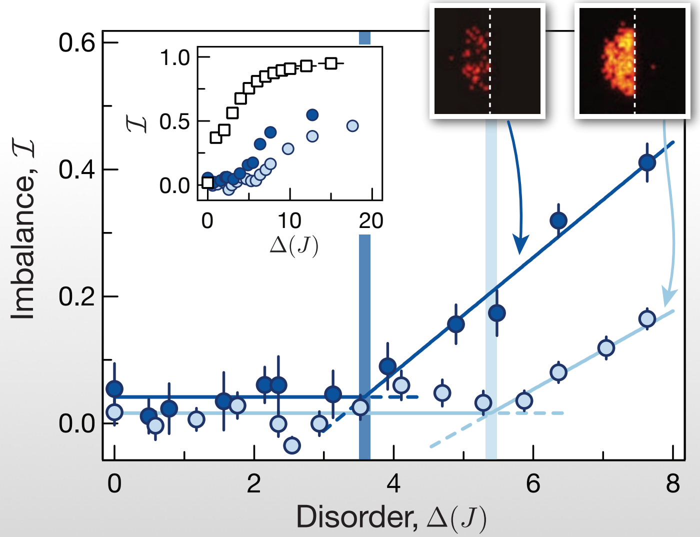
Disorder Effect in Real Space II



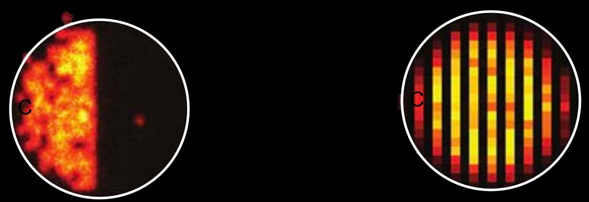
Diverging Length Scale



Role of Interaction

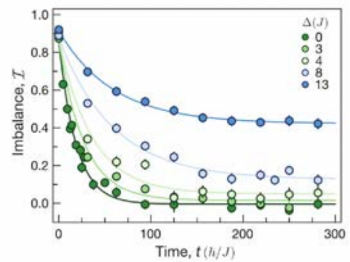


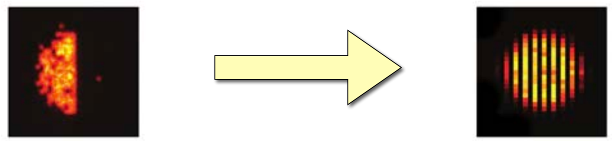
Probing MBL on Different Length and Timescales



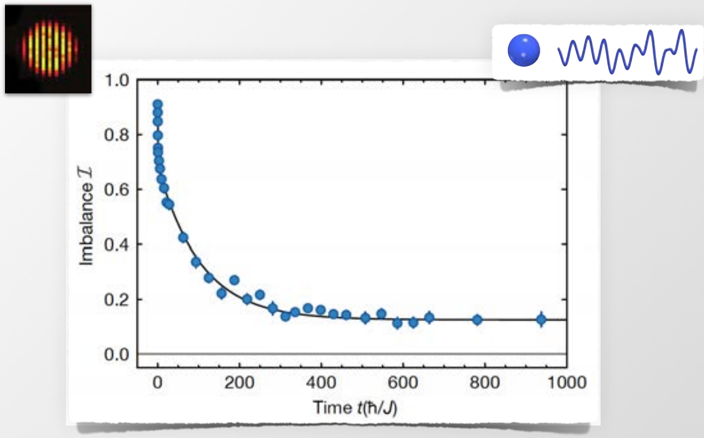
MBL

Better Separation of Timescales





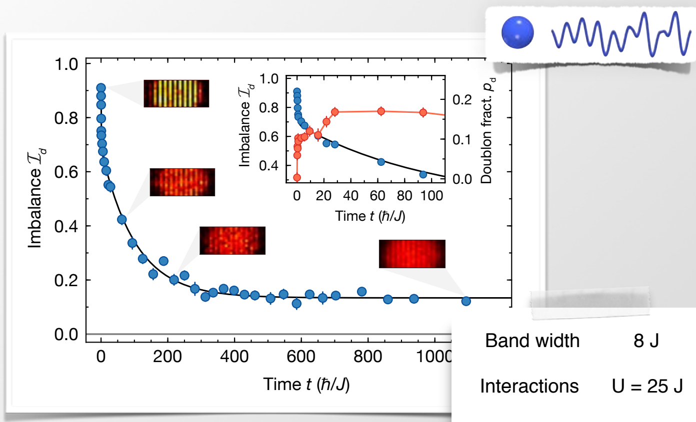
MBL with a small bath — Dynamics without mixture



τ_1~0.6 ħ/J τ_2~105 ħ/J τ_3 > 2500 ħ/J

A. Rubio-Abadal, J-y. Choi et al., arXiv:1805.00056

MBL with a small bath — Dynamics without mixture

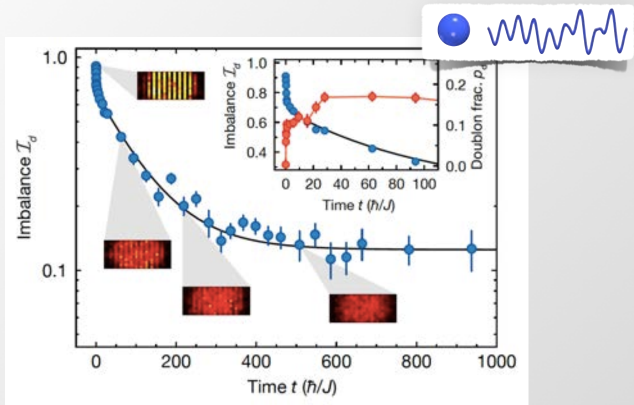


Band width	8 J
Interactions	U = 25 J
Disorder	Δ = 28 J

τ_1~0.6 ħ/J τ_2~105 ħ/J τ_3

A. Rubio-Abadal, J-y. Choi et al., arXiv:1805.00056

MBL with a small bath — Dynamics without mixture



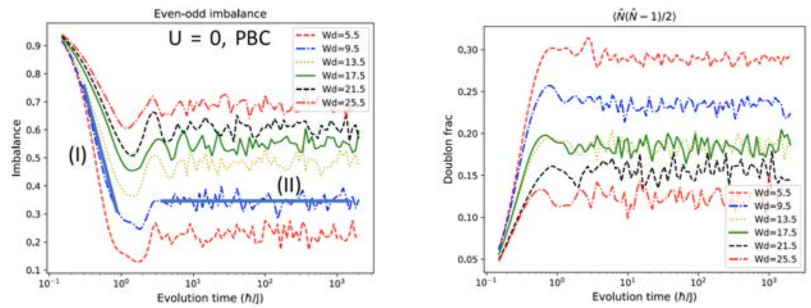
τ_1~0.6 ħ/J τ_2~105 ħ/J τ_3 > 2500 ħ/J

A. Rubio-Abadal, J-y. Choi et al., arXiv:1805.00056

MBL — Numerical Simulation

Non-interacting bosons

(Lx=4,Ly=3, 5 particles)

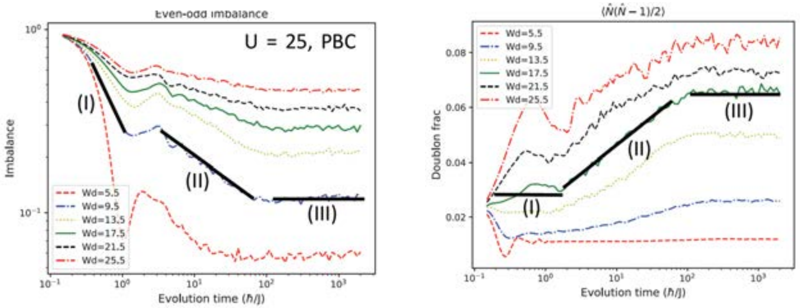


Imbalance **Doublon Fraction**

MBL **Numerical Simulation**

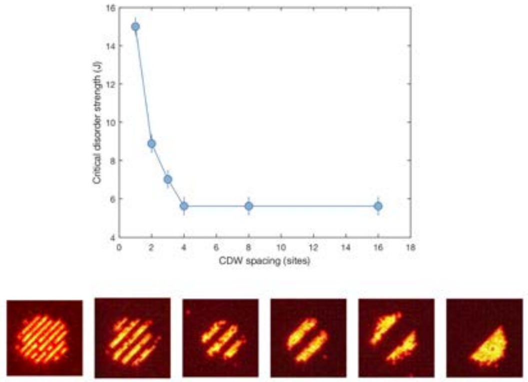
Interacting bosons U/J=25

(Lx=4,Ly=3, 5 particles)



Imbalance **Doublon Fraction**

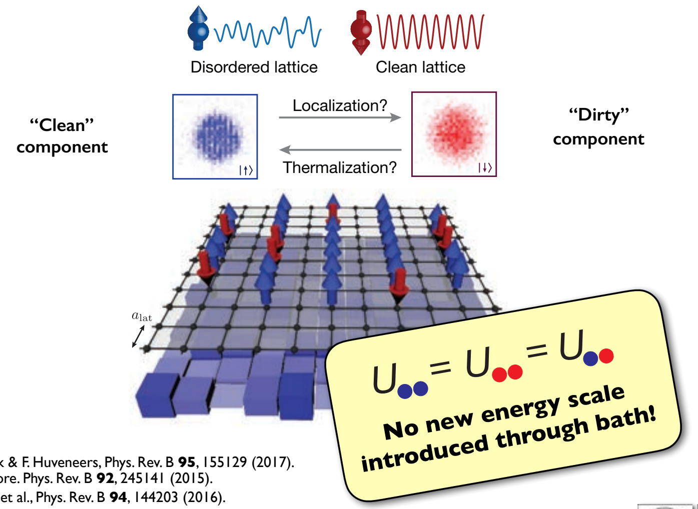
MBL **"Critical Disorder" vs initial State**



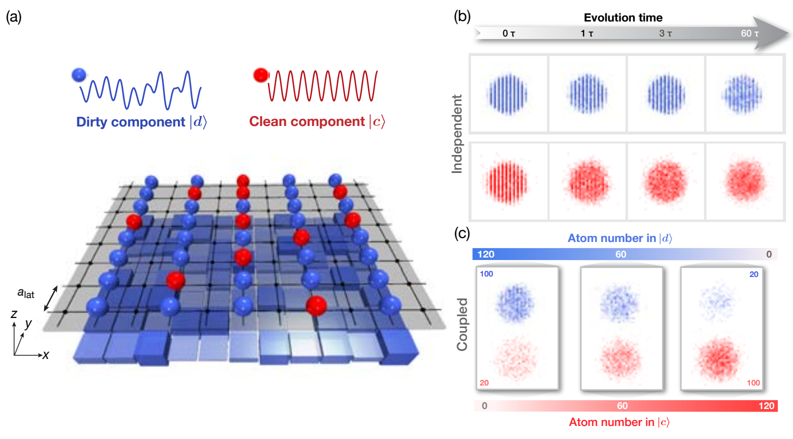
Strong dependence on the observation length scale!

Stability vs Coupling to an Atomic Bath and "MBL Proximity Effect"

Motivation **MBL Coupled to a Finite Bath**



W. De Roeck & F. Huveneers, Phys. Rev. B **95**, 155129 (2017).
R. Nandkishore. Phys. Rev. B **92**, 245141 (2015).
A.Chandran et al., Phys. Rev. B **94**, 144203 (2016).
K. Hyatt et al., Phys. Rev. B **95**, 035132 (2017).
K.Agarwal *et al.*, AdP, 1600326 (2017).



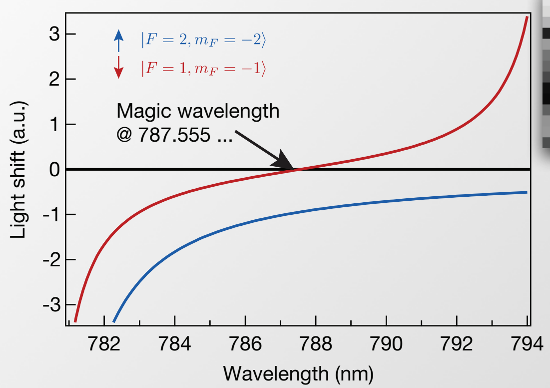
1) Deloc + Loc = Deloc

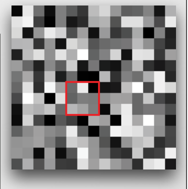
2) Deloc + Loc = Loc

3) Deloc + Loc = Deloc + Loc

Introduction

Spin dependent disorder potential



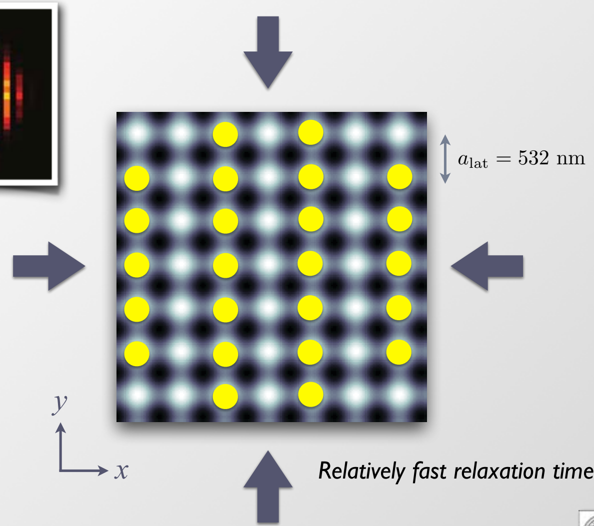


Down spin can be considered as a heat bath

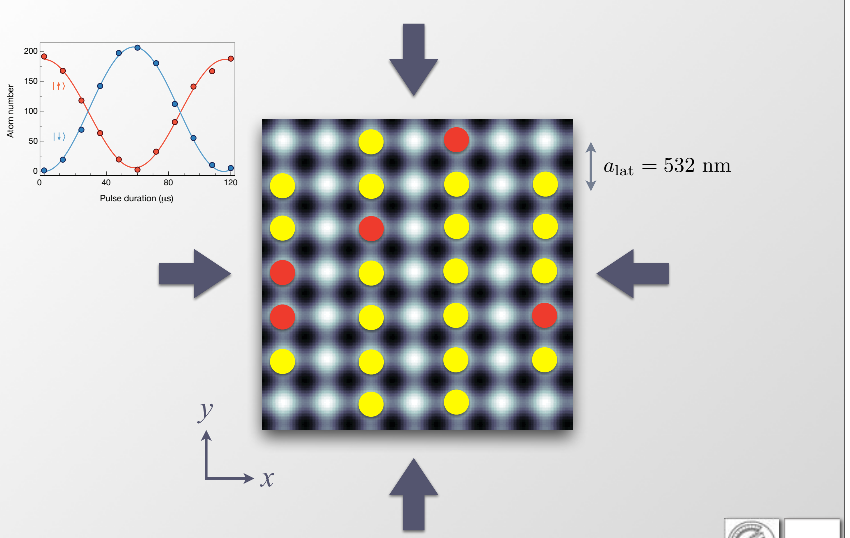
$$U = U_{\downarrow\downarrow} \approx U_{\uparrow\uparrow} \approx U_{\uparrow\downarrow}$$

Introduction

Charge Density Wave

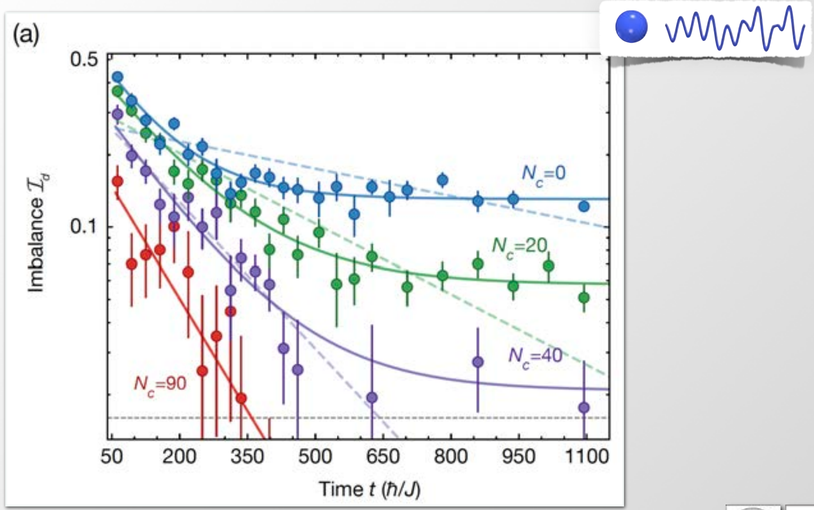


Ergodic spin grains



MBL with a small bath

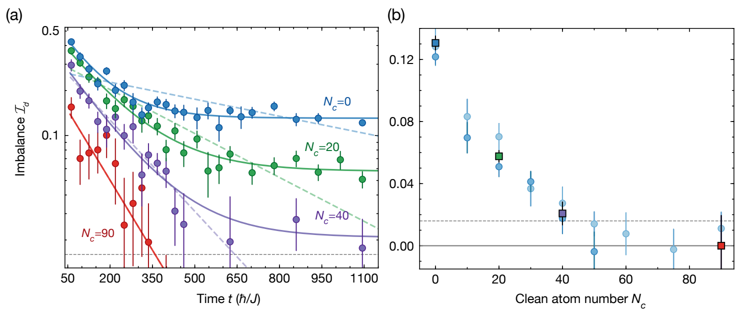
Dynamics of the dirty component



A. Rubio-Abadal, J-y. Choi et al., arXiv:1805.00056

MBL Bath

Stability of MBL vs Size of Bath



Imbalance stable for small admixtures

Larger bath size kills the imbalance ⇨ **Localization non-trivial !**

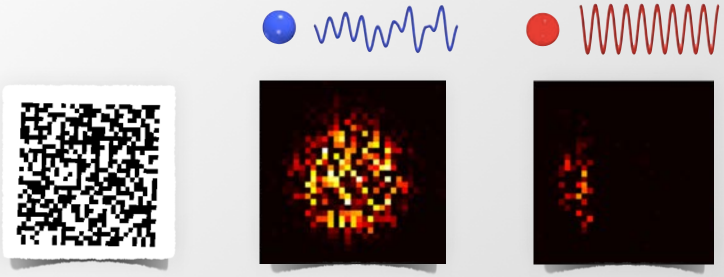
1) Deloc + Loc = Deloc — Larger bath

2) Deloc + Loc = Loc — Small bath

3) Deloc + Loc = Deloc + Loc — Small bath

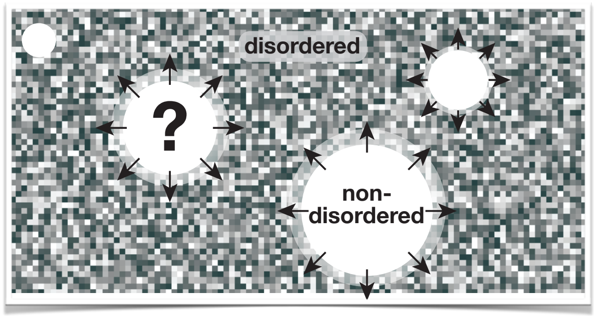
Outlook: proximity effect

Using the atoms to generate a disorder



U. Gavish and Y. Castin, Phys. Rev. Lett. **95**, 020401 (2005)
R. Nandkishore et al., Phys. Rev. B **92**, 245141 (2015)

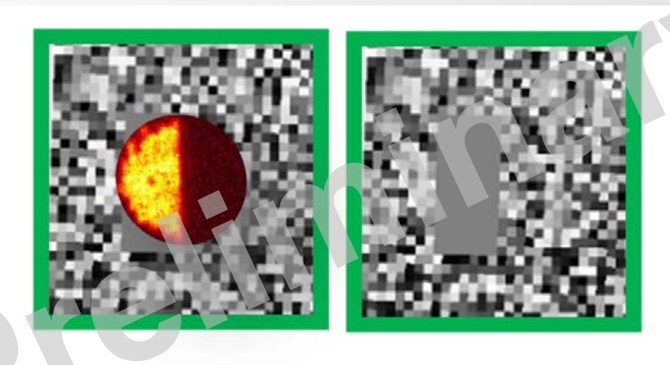
Outlook - Work in Progress



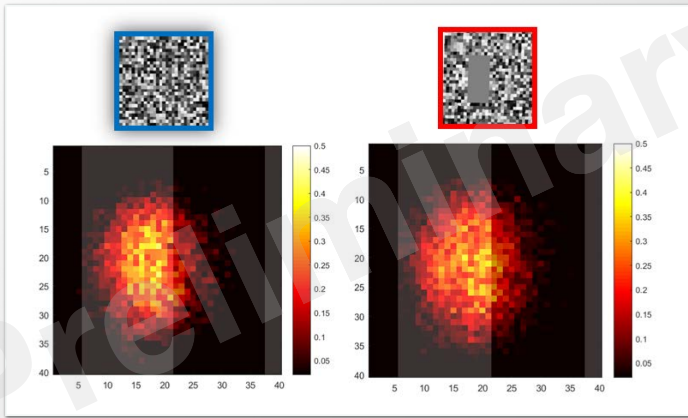
Engineered disorder with **controlled non-disordered (ergodic) grains!**

Avalanches?
Stability?
Range?
Timescales of Instability?
⋮

Engineered Disorder - First Experiments I

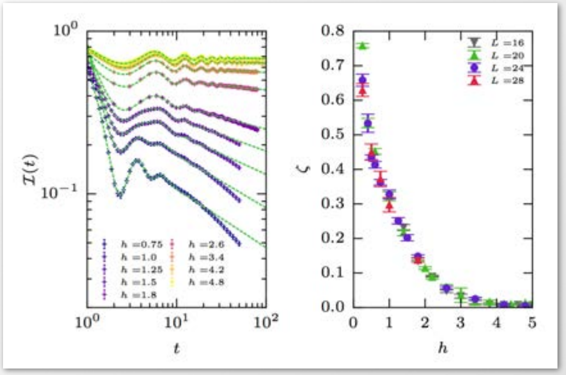


Engineered Disorder - First Experiments I



Relaxation — Slow Dynamics on Ergodic Side

Random field Heisenberg chain



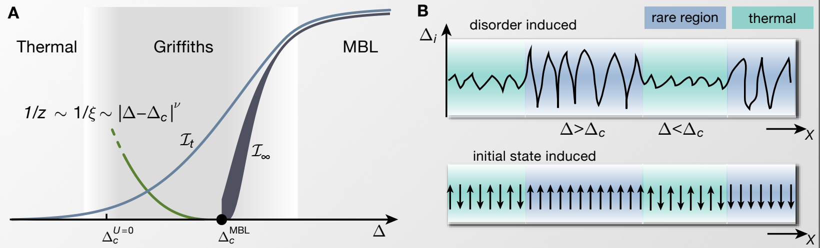
D. Luitz et al. Phys. Rev. B **93**, 060201 (2016)

see also:

R. Vosk, D.A. Huse, and E. Altman, Phys. Rev. X 5, 031032 (2015).
C. Potter, R. Vasseur, and S.A. Parameswaran Phys. Rev. X 5, 031033 (2015).
Y. Bar Lev, G. Cohen, and D. R. Reichman, Phys. Rev. Lett. 114, 100601 (2015).
S. Gopalakrishnan et al.
D. Luitz et al. Phys. Rev. B 93, 060201 (2016)
Phys. Rev. B 93, 134206 (2016).
D. Luitz & Y. Bar Lev, Ann. Phys. 1600350 (2017).
S. Bera, G. De Tomasi, F. Weiner & F. Evers, PRL 118, 196801 (2017).

- Power law decay of imbalance on ergodic side found
- Vanishing exponent at transition!

Relaxation — Experiment - Slow Dynamics on Ergodic Side



Localization Length

$$\xi \propto |\Delta - \Delta_c^{MBL}|^{-\nu}$$

Probability for rare region

$$P(L) \propto e^{-L/\xi}$$

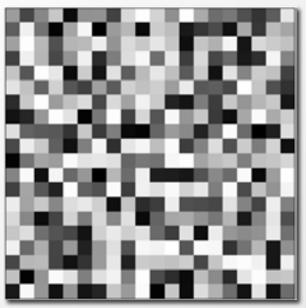
$$\tau \propto e^{L/x_0}$$

Fraction of rare regions that have not thermalised at time *t*

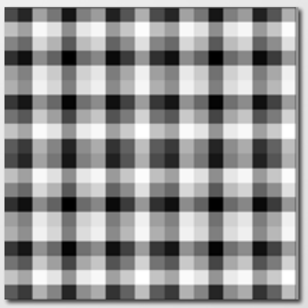
$$I(t) \propto \int_{x_0 \log(t)}^{\infty} P(L)\, dL \propto t^{-1/z} \qquad z = \xi/x_0$$

see: S. Gopalakrishnan et al. Phys. Rev. B 93, 134206 (2016).

Relaxation — True Random Disorder vs Quasi Periodic



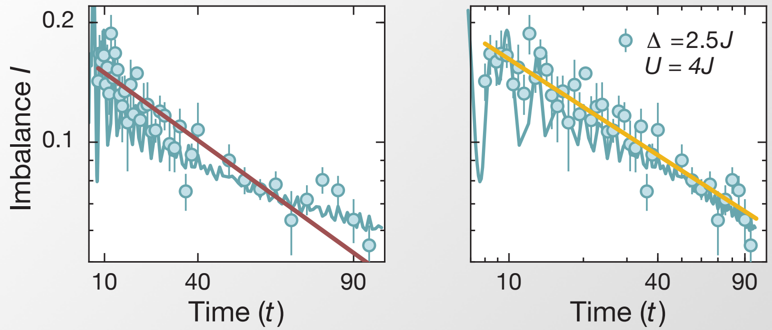
True Random



Quasi Periodic 'Detuning'

See also: Two universality classes for MBL
V. Khemani, D.N. Sheng & D. Huse, Phys. Rev. Lett. **119**, 075702 (2017)

Relaxation — Slow Relaxation Dynamics in Experiments

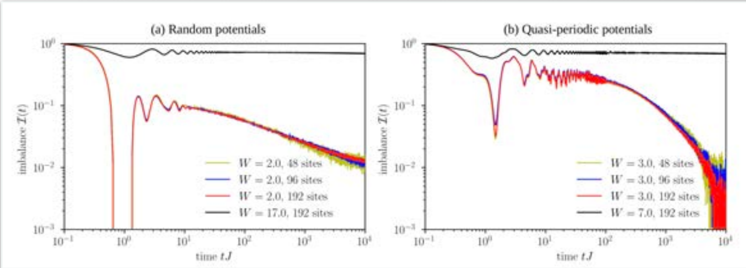


Exponential **Power law**

Relaxation

Relaxation Dynamics - Theory

Power law or non-power law? Conflicting results….



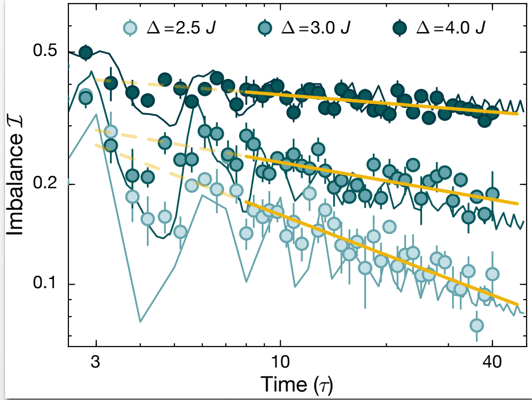
from: S. Weidinger, S. Gopalakrishnan, M. Knap arXiv:1809.02137

see also:

R. Vosk, D.A. Huse, and E. Altman, Phys. Rev. X 5,031032 (2015).
C. Potter, R. Vasseur, and S.A. Parameswaran, Phys. Rev. X 5, 031033 (2015).
Y. Bar Lev, G. Cohen, and D. R. Reichman, Phys. Rev. Lett. 114, 100601 (2015).
S. Gopalakrishnan et al. D. Luitz et al. Phys. Rev. B **93**, 060201 (2016)
Phys. Rev. B 93, 134206 (2016). D. Luitz & Y. Bar Lev, Ann. Phys. 1600350 (2017).
S. Bera, G. De Tomasi, F. Weiner & F. Evers, PRL 118, 196801(2017).

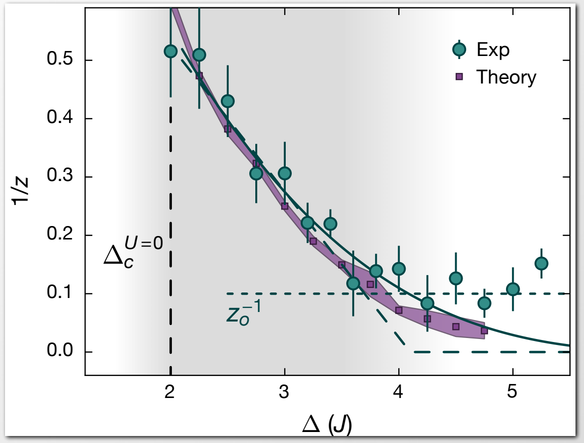
Relaxation

Experiment - Slow Dynamics on Ergodic Side



Relaxation

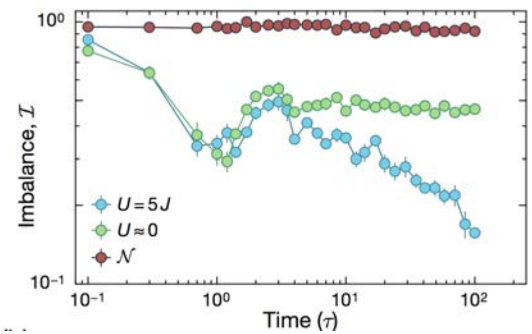
Experiment - Slow Dynamics on Ergodic Side



Relaxation

2D Quasiperiodic

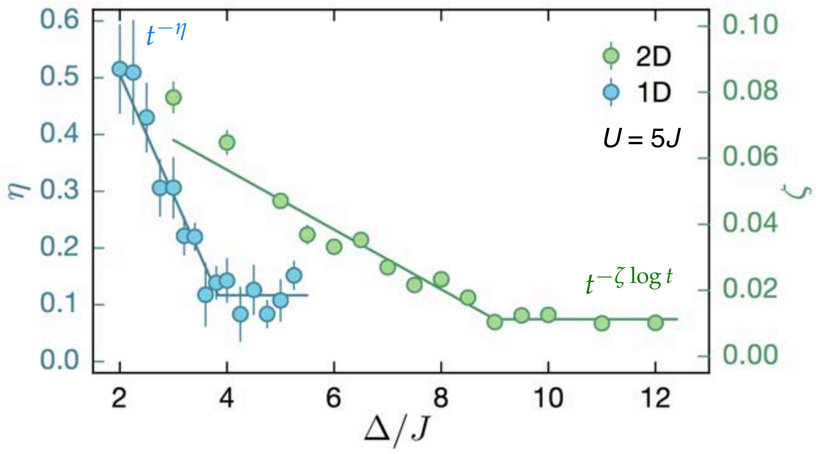
$\Delta = 5J$



Noninteracting localised - Interacting non-localised

Relaxation

1D vs 2D Relaxation Exponents



Big Open Questions

- Stability of MBL
- Nature of Transport in Ergodic Phase
- Definition of Localization Length
- Finite Coupling to Bath

Experimental Advances

- Longer timescales
- Larger systems
- Structured disorder
- Improved isolation from environment