Ultracold quantum systems, Problem set no 4
Magnetic traps and evaporative cooling

Prof. Immanuel Bloch

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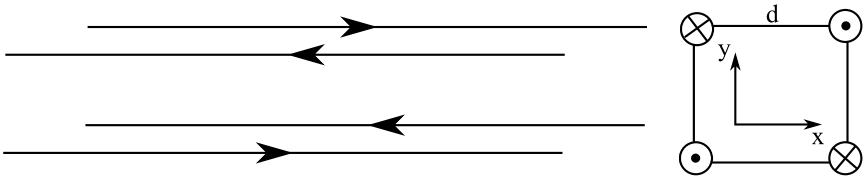
4.1 Magnetic Traps

We look at a "radial" magnetic trap as shown in figure 4.1. The magnetic field is created by four infinitely long wires (say along the z direction with currents on them with equal magnitue I and directions indicated by the arrows. The wires are arranged on a square with edge length d.



(a) Write down the magnetic field vector (in the plane perpendicular to the wires) and sketch it.

(b) now we add a homogeneous magnetic field $\vec{B}_o = B_o \vec{e}_z$ parallel to the wires. Calculate the magnitude of the magnetic field in the (x, y) plane and the minimum larmor frequeny.

(c) Calculate the trap frequency according to this potential. What is the required offset field B_o for adiabatic motion?

(d) For realistic dimensions ($I = 500$ A, $d = 2$ cm, neglecting B_o) estimate the trap depth in the (x,y) plane. What is the depth in 3 dimensions?

4.2 Evaporative Cooling

Let N be the total atom number in a harmonic magnetic trap. $E = 3Nk_BT$ is the total energy, T the temperature. $n = N/V_e$ is the (average) atomic density. In equilibrium, the effective volume V_e of the trap is (by the equipartition theorem) $V_e = \left(\frac{2\pi k_B T}{m\omega^2}\right)^{3/2}$. The relative atomic velocity for a single component gas turns out to be $\bar{v}_r = \sqrt{2}\bar{v}_{th}$.

4.2.1

(a) Assuming the trap depth ϵ_{tr} is fixed and finite, calculate the temperature as a function of time.

(b) Now assume the trap depth to track the temperature: $\epsilon_{tr} = \eta k_B T, \eta \gg 1$. Prove that $\frac{\dot{D}}{D} = (3 - \eta)\frac{\dot{N}}{N}$. Here $D = n\lambda_{th}^3$ denotes the phase space density and λ_{th} is the thermal DeBroglie wave length.

(c) What is the minimum value for the evaporation parameter $\eta = \epsilon_{tr}/k_BT$ to observe run-away evaporation?

4.2.2

The life time of ultracold gases is limited by the quality of the vacuum system and amounts to typically 1 minute in the collision less regime. This means that evaporative cooling to the desired temperature should be completed within typically 15 seconds. Let us consider the case of ^{87}Rb, which for $T < 500\mu$K, has a scattering cross section given by $\sigma = 8\pi a^2$, with $a \approx 100a_0$ ($a_0 = 0.529 \times 10^{-10}$ m is the Bohr radius). The trap frequency is $m\omega^2/k_B = 1000$ K/m^2.

(a) Calculate the density n_0 for which the evaporation rate is $\dot{N}/N = -1$ s^{-1} at $T = 0.5$ mK and evaporation parameter $\eta = 5$.

(b) What is the thermalization time under the conditions of question (a)?

(c) Is the gas collisionless or hydrodynamic under the conditions of question (a)? Hint: A gas is called collisionless if the mean free path l is much larger than the size (volume V) of the trap ($l \gg V^{1/3}$) conversely if it is much shorter, the gas is called hydrodynamic ($l \ll V^{1/3}$)

4.3 Relative coupling strengths (optional)

From the lecture it is known that the relative couplings between different magnetic substates are only determined by Clebsch-Gordan coefficients. This (surprising) result is a general feature of any Tensor coupling and is summarized in the **Wigner-Eckart theorem** (see e.g. Messiah, Quantum mechanics):

Let $\vec{\mathbf{T}}^{(k)}$ be a irreducible tensor operator of rank k ($k \geq 0$ integer), and $|nlm\rangle$ eigenstates of the systems total angular momentum $\mathbf{L}$ with eigen values $l(l+1)$ and m of L^2 and L_z respectively and n being the rest of the required quantum numbers. Then

$$\langle n'l'm'|\, T_q^{(k)}\, |nlm\rangle = \langle lk; l'm'|\, lmkq\rangle\, \langle n'|\, |T^{(k)}|\, |n\rangle \tag{1}$$

where $\langle n'|\,|T^{(k)}|\,|n\rangle$ does *only* depend on n' and n and the operator $\mathbf{T^{(k)}}$ and $\langle lmkq|\, lk; l'm'\rangle = \langle lk; l'm'|\, lmkq\rangle$ are the Clebsch-Gordan coefficients introduced on problem set 1. Obviously q can vary in integer steps between $-k$ and k. The precise definition of an irreducible tensor operator can be found in the literature.

4.3.1 selection rules

The dipole operator $\vec{\mathbf{d}} = e\vec{\mathbf{r}}$ operator is a "vector" operator, i.e. a tensor operator of rank one.

(a) Relate the dipole selection rules to the properties of the clebsch gordan coefficients.

(b) Relate the polarizations $\{\sigma^+, \sigma^-, \pi\}$ to the components q of the tensor.

(c) Do you have an idea, to which polarization states of the light these components correspond to?

4.3.2 (for those who like lengthy number crunching)

The transition probability for a given intensity of the light field is proportional to the square of the dipole matrix element. Calculate the relative transition probabilities for an $F = 0$ to $F = 1$ transition.