Ultracold quantum gases
Problem set 5

Prof. Immanuel Bloch
(Due Nov. 23, 2009)
Questions: ask Simon (simon.foelling@lmu.de)

5.1 Expansion of a BEC

In the following we will look at how a very cold atom cloud, which is released from it's trap and expands. You have already heard in the lecture that this is a very common thing to do in order to make a measurement on cold gases. Probably more than 50% of all scientific results from BEC-type experiments are obtained with some sort of time-of-flight expansion, where the density distribution is analyzed after the cloud has expanded without external forces for a specific time t_{TOF}. You also have seen the famous pictures of the emerging BEC during the condensation. Here we will look at how these pictures are formed. It's not magic at all, and in the following it is divided into small parts. The important sections of part 1, 2 and 3 are independent from each other though.

As is typically the case, we assume that the BEC is contained in a parabolic trap which follows the form $V(r) = 1/2 \cdot m\omega_T^2 r^2$. m is the mass of the atoms.

5.1.1 Thermal gas

Firstly, let's assume the cloud is cold, but non-condensed, the temperature is $T \gg T_c$. Since the interactions can be neglected (except for the thermalization), the shape is atom number independent.

- What is the shape of the cloud inside the trap? You should not think about trap eigenstates here (although you can do it that way), it is straightforward if you think about a classical gas. With the correct arguments regarding the occupation (atom density) in available states, you only need the Boltzmann distribution for this.
- How does the shape of the cloud evolve if the trap is suddenly removed? One way to do this is to consider how the atoms from each point in space will distribute over time.
- At which point in time (in units of the trap period $2\pi/\omega_T$) has the size of the cloud doubled?

5.1.2 Non-Interacting BEC

Let us assume now that the gas is partially condensed; there is a BEC fraction of non-interacting atoms as well as a thermal fraction present. Let us assume Rubidium again, so $m = 87\,\mathrm{u}$. The trap frequency is $\omega_T = 2\pi \times 30\,\mathrm{Hz}$. We have 40000 atoms, the temperature is 140 nK. For now, no interactions are present (except that the cloud inside the trap is fully equilibrated).

- From lecture: what is the condensation temperature T_c? How many atoms are in the condensate part?
- You can look at this problem by considering each axis separately. Give a reason why, from the Hamiltonian.

  What is the shape of the condensate part in the trap? *(no interactions, simple!)*
- Without the interactions, how does the shape of the condensate part evolve when expanding freely? *(One possible way is to express the initial state in free-space eigenstates, and evolve these in time, then calculate the resulting density distribution).*
- Plot the total distribution (BEC and thermal part), for t_{TOF} =0, 5, 10, 20 ms

- * If the trap frequencies along two directions of space (ω_x and ω_y) are not identical, what is the shape inside the trap? In which direction is it largest? *(simple!)* How does it change during expansion? *(also simple!)*

5.1.3 Interacting gas

Now we consider the real thing: The atoms interact, both in the trap and while flying free. After T_{TOF} a picture of the density distribution is taken. Amongst other things, such pictures then lead to the Nobel prize of 2001.

You already know the in-trap part from the lecture. The interactions will of course also affect the motion of the cloud while it is expanding. So you would expect a very complicated behaviour of the cloud - but surprisingly, it turns out to be very simple! You can see why, with a very simple semiclassical approach:

In the situation where Thomas-Fermi-distribution applies, the gas is mostly dominated by the interaction energy inside the system, the kinetic parts can be neglected for the condensed fraction. As you saw in the lecture, for one individual atom, the density distribution of all the other atoms is treated as an effective potential. After the trap has been removed, this is the dominant part of the Hamiltonian that is left, therefore it will determine the kinetics.

- After switch-off of the trap: What is the shape of the effective potential each atom experiences? Assume a general Thomas-Fermi radius R.

- The whole system is obviously spherically symmetric, and will stay that way whatever happens. Therefore you can solve the problem considering only one dimension. What is the force on each individual atom at a given location r in space due to the potential? What is the acceleration expressed in r, and which direction does it point? If all atoms start to move with their respective acceleration, what happens to the shape of the distribution in this particular case? *(It is easy to see if you assume for a moment that the acceleration is constant for each individual atom)*

- Assuming a Thomas-Fermi (TF) profile with initial TF radius R_0 and interaction parameter g, give the full expression for the shape of the cloud after a given time t_{TOF}, using the *real* accelerations *(hint: knowing already the general shape from the previous question, one way is to just look at one point of the distribution, such as the outer edge, using a simple differential equation for velocities).*

- * Argue that the same approach also works if the original confining potential is harmonic in all directions, but with different frequencies (Easy if you look at the geometry of accelerations). What is the ratio of TF radii of the cloud along two directions with two different frequencies (ω_x and ω_y) before expansion? Is this ratio constant when expansion starts? Figure out the expansion in this case for arbitrary times and compare to the non-interacting case.

- Put it all together: Plot the distribution for the trap given above (all frequencies the same with $2\pi 30\,\mathrm{Hz}$, with 40000 atoms, temperature T, the interaction constant $g = 4\pi\hbar^2 a_s/m$, and $a_s = 100\,$Bohr radii for Rubidium. Plot three different times t_{TOF} =0, 10, 20 ms for $T = 140\,\mathrm{nK}$. In addition, vary the Temperature: Plot the distribution for t_{TOF} =20 ms for T =140, 200, 400 nK