

Ultracold quantum gases

Problem set 12

Prof. Immanuel Bloch
(Due Jan. 25, 2010)

Questions: ask Simon (simon.foelling@lmu.de)

12.1 Scattering in the shell potential

Let's consider the scattering of two particles with a hollow shell potential

$$V(r) = U\delta(r - a),$$

so a is the radius of the shell and r the interparticle distance.

Obviously, due to the delta function the repulsive potential has an infinitely large peak, so this is like a „wall“ to the outside and the system resembles very much the hard sphere potential discussed in the lecture. But not quite.

12.1.1

Calculate the s -wave phase shift of this potential as a function of U , for the low-energy limit ($l = 0$ and low k).

12.1.2

Under which conditions does this system actually behave the same way as the hard sphere potential, as speculated?

12.1.3

Even if the constant U is very large, the potential can have very different properties from the hard sphere. Under which circumstances does this happen?

12.2 Scattering orbitals

As you learned, for low energies the higher order scattering channels „freeze out“ until only the s -wave scattering remains. This is almost always true for the kind of experiments we discuss - however, only almost always. There are exceptions. Use anything you have learned so far and give at least three situations where it is not just s -wave scattering that is going on.

(Increasing the temperature scale so that *all* channels (not just specific ones) contribute does not count; and also, only short range potentials are allowed!)

In each case, describe exactly the criteria that are necessary for your scheme to work.