

Ultracold quantum gases

Problem set 9

Prof. Immanuel Bloch

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Questions: ask Simon (simon.foelling@lmu.de)

9.1 Interaction of two tightly confined atoms

We are considering two atoms confined in the potential minimum of an optical lattice. The confinement is very strong, therefore: It can be assumed that the particles „see“ only the parabolic part of the potential very close to the minimum. Secondly, the wave functions are so confined that the opposite regime to the Thomas-Fermi regime applies: The wave functions are dominated by the kinetic and potential terms of the hamiltonian, and the interaction can be neglected for the shape of the wave functions. All this makes the determination of the wave function shape very easy.

9.1.1

For an interaction parameter $g = 4\pi\hbar^2 a_s/m$, and $a_s = 100$ Bohr radii for Rubidium, calculate the total energy component of the interaction term of the Schroedinger equation for two atoms sitting together in the same site of a lattice with periodicity a_{lat} and total depth V_{lat} , so the lattice potential is $V(x) = V_{lat} \cos(2\pi x/a_{lat})$.

9.1.2

Plot the energy you determined in units of kHz (by dividing the energy by h) for a lattice periodicity $a_{lat} = 420$ nm, against the lattice depth V_{lat} (range 0 to $h \times 150$ kHz).

9.1.3

Since we made several approximations in the beginning: Discuss under which conditions the calculation is valid and when it will break down.

9.2 Dispersion relation, group velocity, Bloch oscillations and all that...

In the lecture you have learned about the dispersion relation for a particle in a lattice. The dispersion relation is the relation between momentum and (kinetic) energy of a particle. In the general case, for a lattice it is evaluated numerically, but you were given an approximation for the tight-binding case of a deep lattice:

$$E_q = -2J \cos(\pi q/k_{lat}) + const.$$

Here, q is the lattice (quasi-)momentum of the particle, and the assumption is that the lattice is produced by light which is retro-reflected with a wavelength of $\lambda_{lat} = 2\pi/k_{lat}$. Now, still the constant J needs to be determined, which is usually done numerically, but this constant now characterizes the lattice.

A good approximation is the following formula:

$$J \approx \frac{4E_r}{\pi} \left(\frac{V_{lat}}{E_r} \right)^{3/4} \exp \left(-2\sqrt{V_{lat}/E_r} \right),$$

where the recoil energy is defined as

$$E_r = \frac{\hbar^2 k_{lat}^2}{2m}$$

with k_{lat} the k -vector of the lattice light and m the mass of the atom (You should only plug in this formula when you really have to, otherwise keep J).

Bloch oscillations happen when a particle is accelerated to the edge of the Brillouin zone of the lattice, and gets reflected to the opposite edge. In the photon picture which you saw in the lecture, this means that the atom absorbs a (lattice) photon coming from the front and emits it in the opposite direction (into the opposite beam), hence receiving a total momentum displacement of $2\hbar k_{lat}$.

9.2.1

Obviously, Bloch oscillations are periodic. Assuming a lattice with wavelength λ_{lat} and a constant external force F in positive direction acting on the atom (for example gravity), what is the periodicity (or frequency) of the Bloch oscillations? Does it depend on lattice depth (and therefore on J)?

9.2.2

The Bloch oscillation is a periodic trajectory in momentum space. However, this of course means there is also (periodic) motion in real space. (One other picture of Bloch oscillations is that the particle moves through the periodic lattice potential until its speed and therefore wavelength is such that the Bragg condition for reflection is fulfilled – hence the atom gets reflected backwards.)

Of course we all know that for the center-of-mass motion of particles in non-trivial potentials the *group velocity* matters, not just the quasimomentum (that is why it is called quasi after all). The group velocity is given by the derivative of the kinetic energy by the quasimomentum (look it up in your favourite QM textbook in case you forgot). Using this knowledge and the known dispersion relation: Derive an expression for how far (in units of lattice sites) does the atom actually move during one Bloch oscillation? Does this distance increase with the accelerating force? What is the shape of the trajectory (position vs. time)? Does the range of the motion increase or decrease with the lattice depth (For this you need to look at the formula for J)? Why is this so?

Calculate the motion distance for the following realistic (for a recent experiment) parameters: $m = 2.21 \times 10^{-22}$ kg, $\lambda_{lat} = 1064$ nm, $V_{lat} = 7.9 E_r$, $F = m \times g$ (gravity). Discuss the (surprising) result!