Ultracold quantum gases
Problem set no 3

Prof. Immanuel Bloch
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1 Two level atom: Landau Zener Sweep

We consider a two-level atom whose levels are coupled by a laser field with Rabi frequency Ω_0 and detuning δ. As was shown in the lecture, the Hamiltonian of this system can be written in the Rotating Frame as:

$$\hat{H} = \frac{1}{2}\begin{pmatrix} -\delta & \Omega_0 \\ \Omega_0 & \delta \end{pmatrix}$$

- Diagonalize this Hamiltonian.
- Plot the eigenenergies and the ground state contribution of one of the eigenstates as a function of detuning for $\Omega = 0.1$ and $\Omega = 1$.
- What happens to a ground state atom, if the detuning -at constant Rabi frequency- is ramped very slowly from large negative to large positive values?
- Can you think of an estimate to when a ramp is "slow"?

2 Magneto Optical Trap (MOT)

In Doppler cooling the force on the atom is given by the velocity-dependend radiation pressure of two counterpropagating laser beams that are detuned by a detuning δ from the atomic resonance.

2.1 Capture velocity

- Describe in your own words why the radiation force has a maximum at a specific velocity.

- This velocity is called the *capture velocity* and is often used as a simple criterion to evaluate what fraction of a thermal gas can be cooled by Doppler cooling. Calculate the capture velocity for a gas of ^{87}Rb atoms ($\lambda = 780.24$ nm, $\Gamma = 6$ MHz, $m = 87 \cdot 1.673 \times 10^{-27}$kg) and detunings $\delta = 1\Gamma$, 3Γ.
- In addition, calculate the captureable fraction of a thermal gas at $T = 300$ K by assuming that all atoms slower than the capture velocity are captured while all faster atoms are not captured. Hint: At these temperatures the atomic distribution can be treated classically.

2.2 Rate equation

The easiest model for the loading dynamics of a MOT is to assume a constant loading rate L [atoms/s] and a fixed lifetime τ [s] of a laser-cooled atom in the MOT. This assumes that the lifetime is given by collisions with the (room-temperature) background gas.

- Write down the associated rate equation and solve it.
- How does the final atom number in the MOT depend on the pressure of the background gas?

2.3 Time of Flight Imaging

The standard method for measuring the temperature of a cold/ultracold atomic cloud is the so-called *time-of-flight* imaging method. In this method all laser beams and magnetic fields are suddenly switched off and the atoms are imaged after a finite time of free flight.

- Assume the initial density distribution (in 1D) of the atoms to be Gaussian with width w_x. Calculate the density distribution after a time t for atoms with an initial temperature T.
- Sketch the width of the density distribution as a function of time and discuss the easiest way to extract the temperature of the initial cloud from the width.

If you have any questions write an email to ulrich.schneider@lmu.de or stop by in room H210.