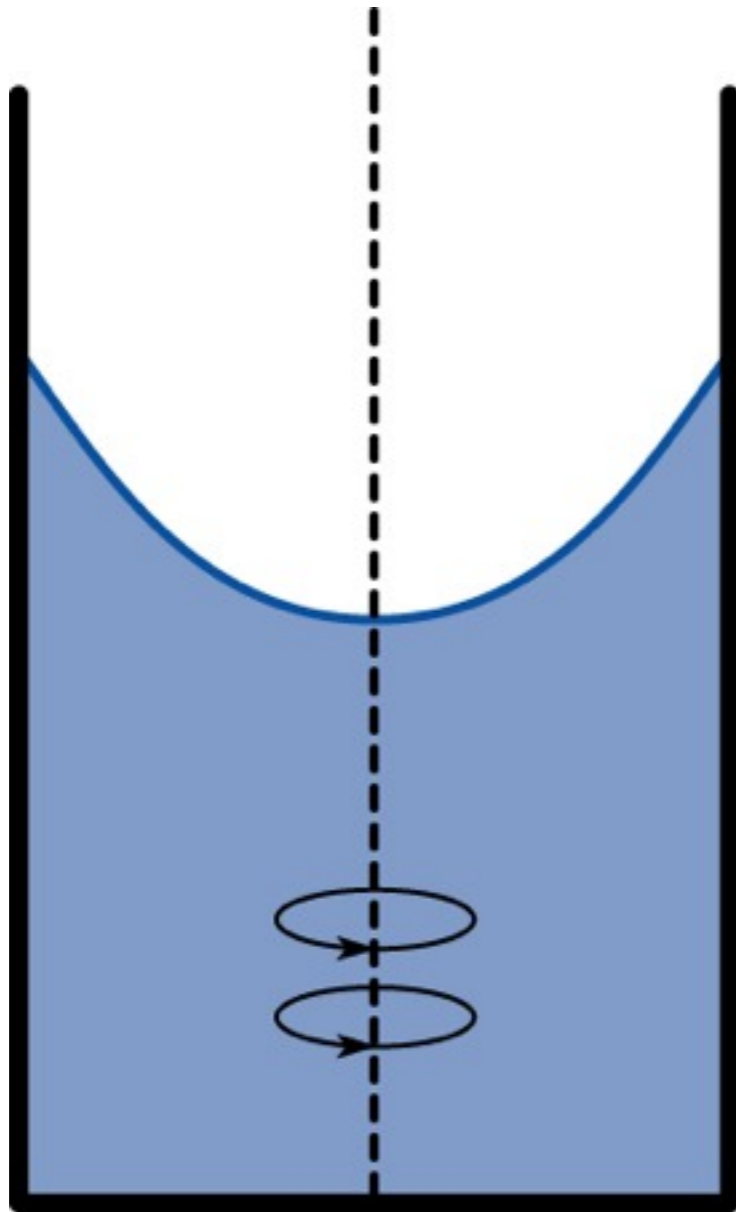


# *Rotating a Classical Fluid*

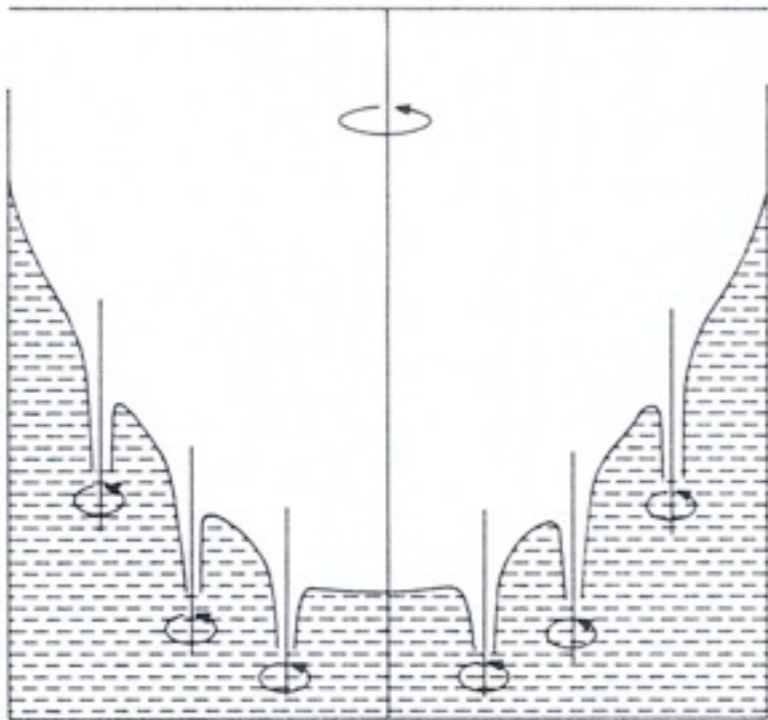
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Surface of the fluid is shaped like a parabola !

# Rotating a Quantum Fluid

e.g. Rotating bucket experiment in He II

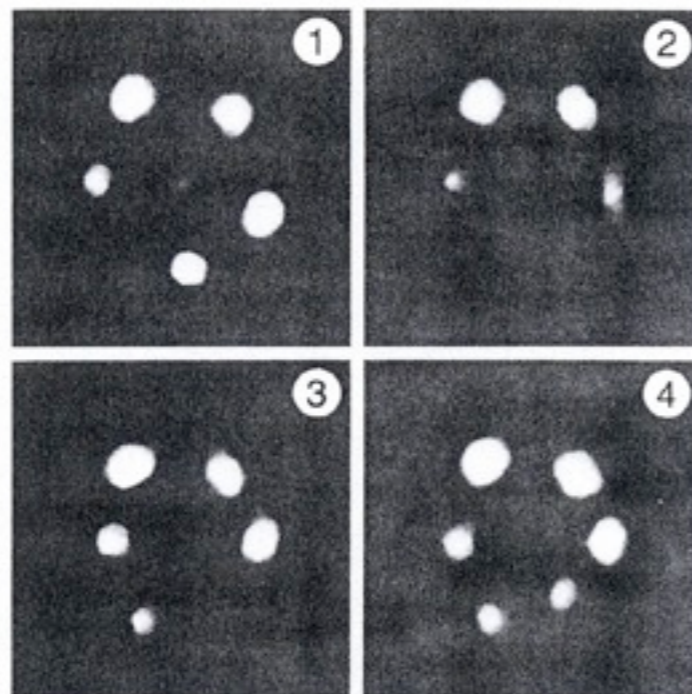


Below a critical angular velocity the quantum fluid remains in **rest**.

Above a critical angular velocity one or more **singular lines** appear in the fluid.



The **circulation** around such singular lines is **quantized**.



These **vortex lines** can be detected via **trapping of electrons** on the **vortex cores** !

Yarmchuk and Packard (1982)

# Quantization of Circulation

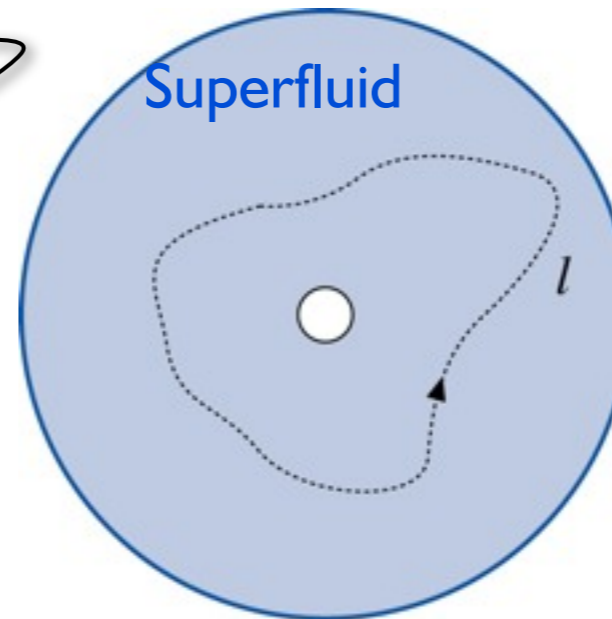
Definition of Circulation:

$$\kappa = \int \mathbf{v} d\mathbf{r}$$

$$\mathbf{v} = \frac{h}{m} \nabla \theta(\mathbf{r})$$

Velocity field of the order parameter

$$\kappa = \frac{h}{m} \Delta \theta$$



Superfluid occupying a multiply connected region. Contour path  $l$  within the superfluid.

The order parameter has to be single valued !

$$\hookrightarrow \Delta \theta = n \cdot 2\pi$$

$$\kappa = n \cdot \frac{h}{m}$$

Circulation is quantized !

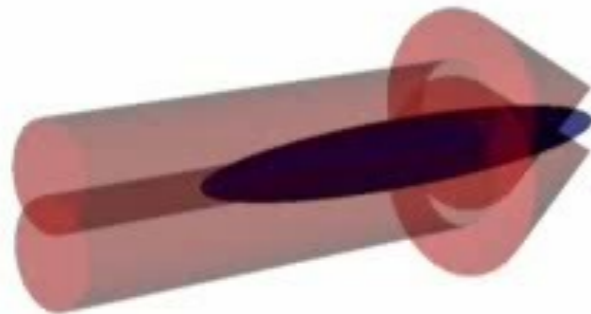
# Stirring a Bose-Einstein Condensate

Magnetic trapping potential:  $V(r) = \frac{1}{2}m(\omega_\rho^2 x^2 + \omega_\rho^2 y^2 + \omega_z^2 z^2)$

Use light beams to create a deformation of the potential that can be rotated !

$$V_{dip}(r) = \frac{1}{2}m(\varepsilon_X \omega_\rho^2 X^2 + \varepsilon_Y \omega_\rho^2 Y^2)$$

X,Y basis is rotated with an angular velocity  $\Omega$ .



Hamiltonian in the co-rotating frame

$$H_{rot} = H - \Omega L_z$$

Vortex state

raises energy

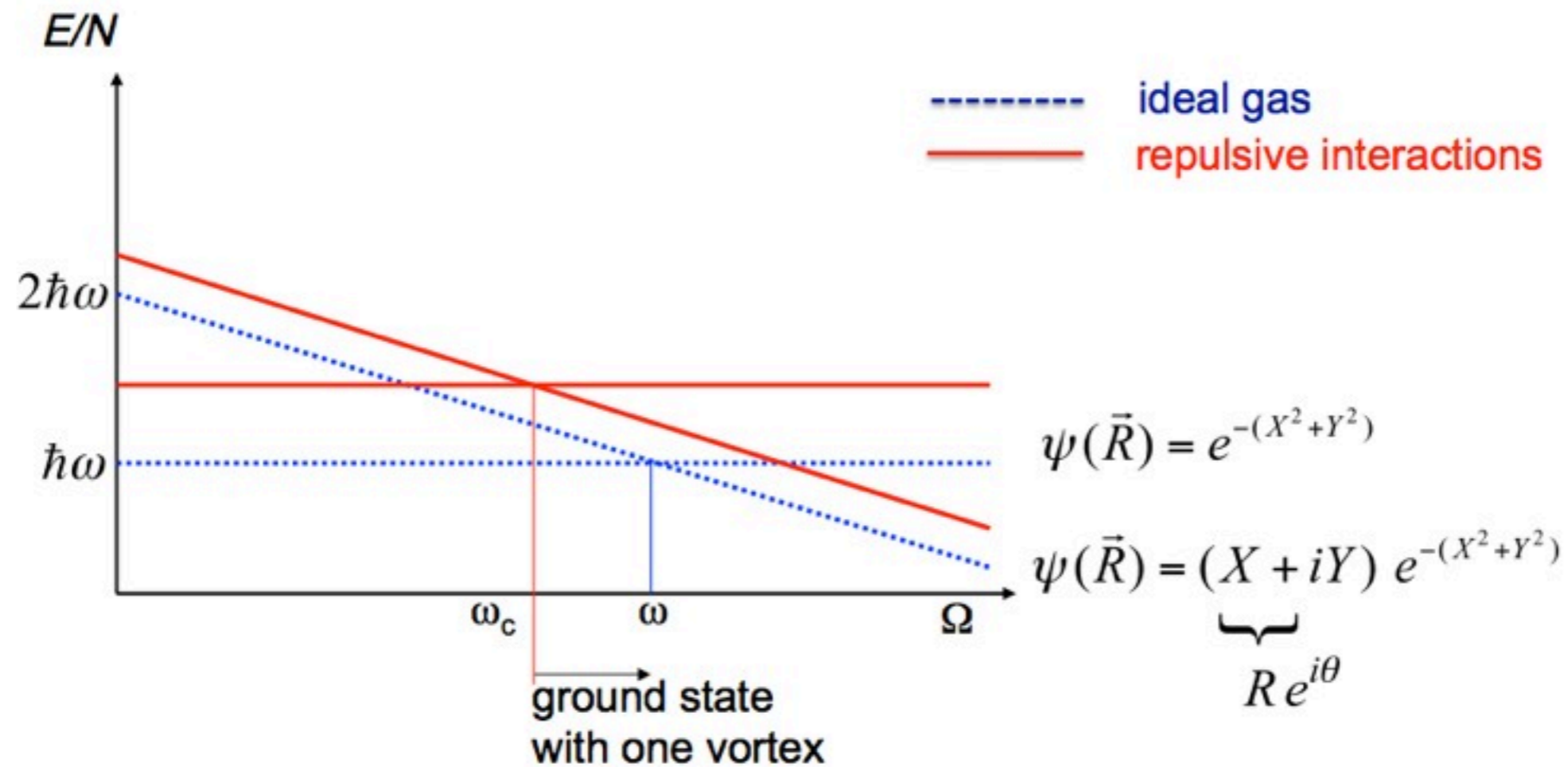
Vortex state

lowers energy

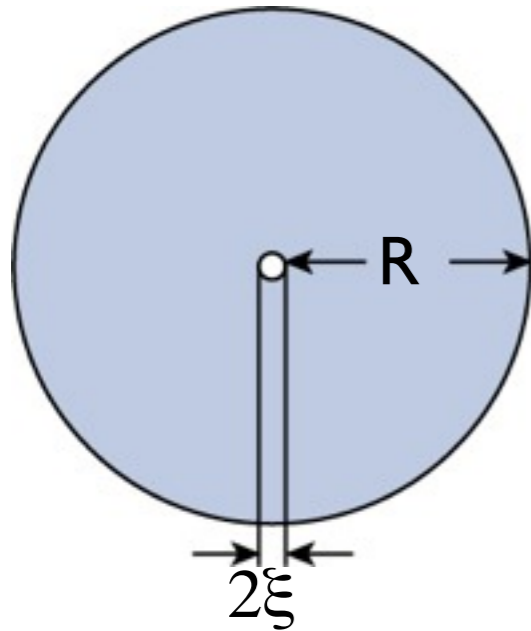
# Vortices as a Ground State of the New Hamiltonian

Neglect for simplicity the term in  $\varepsilon$

single-particle Hamiltonian in rotating frame:  $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 r^2 - \Omega L_z$



# Vortices in a Gaseous BEC

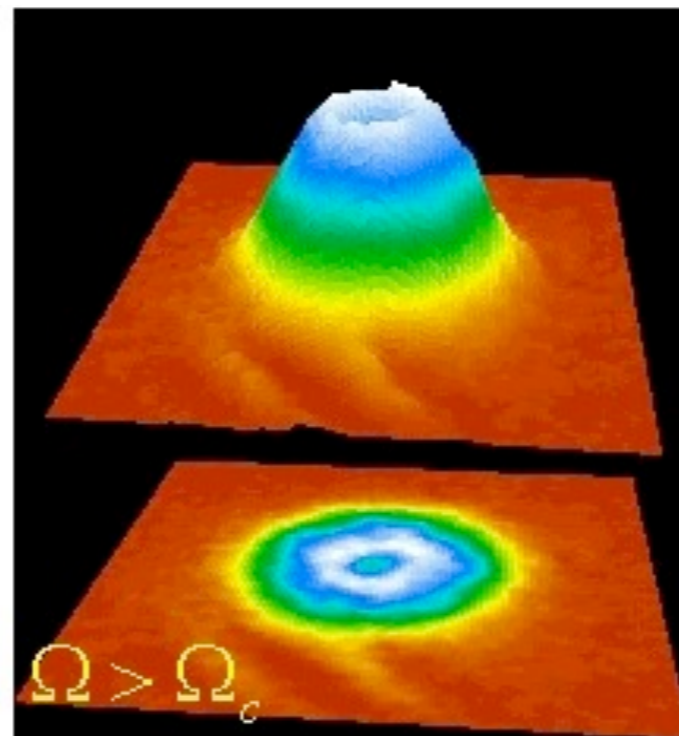
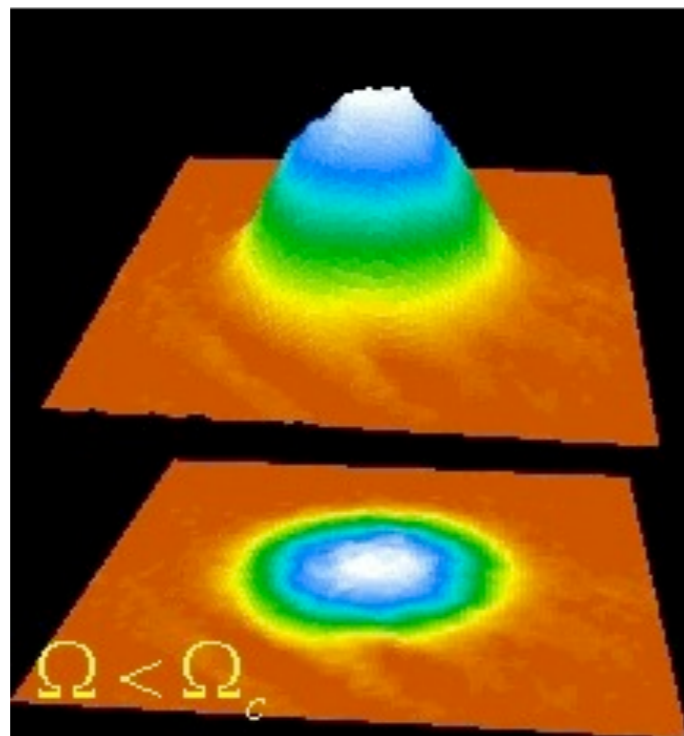


Vortex core size is given by the **healing length**:

$$\xi = \frac{1}{\sqrt{8\pi n a}} \approx 200 \text{ nm}$$

Too small to be imaged directly in the trap !

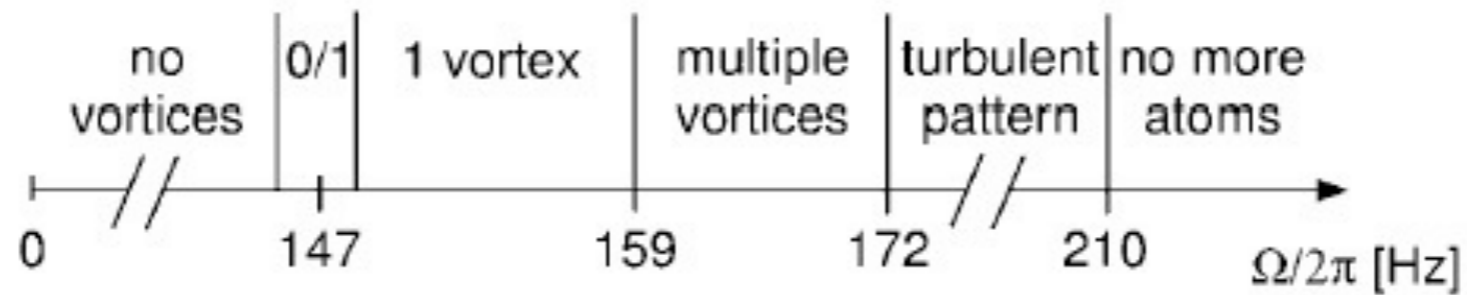
**Solution:** Use TOF expansion such that all dimensions are expanded by a factor  $\sqrt{1 + \omega_p^2 T^2}$



K. W. Madison  
et al. (2000)

# Abrikosov Vortex Arrays (1)

Phase diagram of vortex nucleation



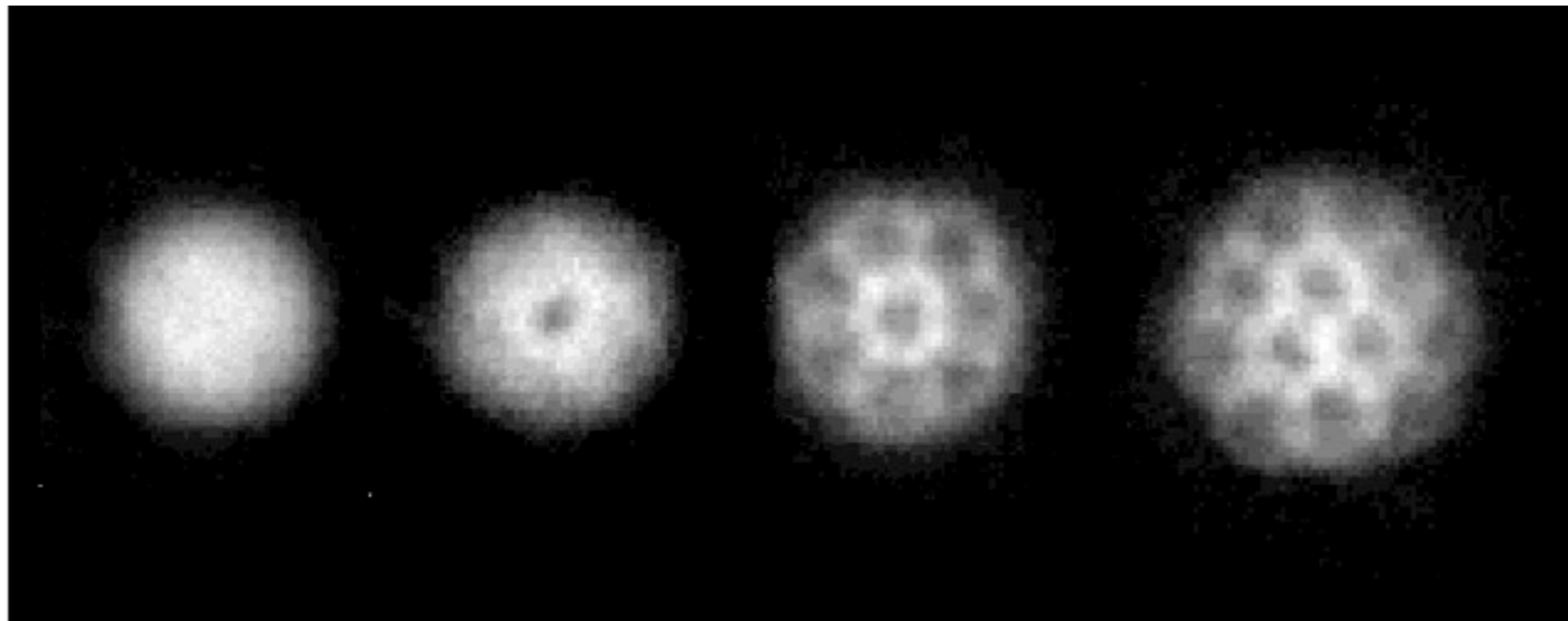
# of vortices:

0

1

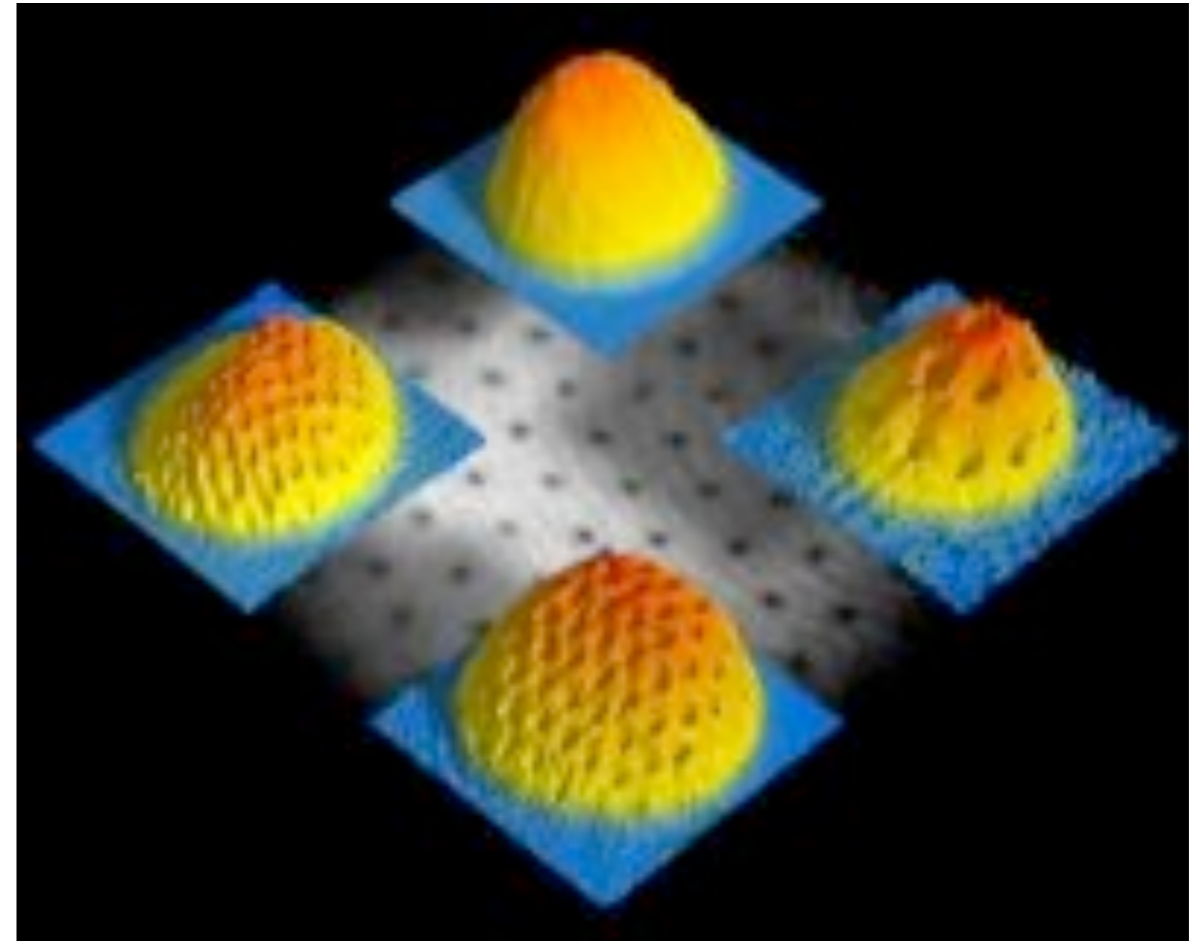
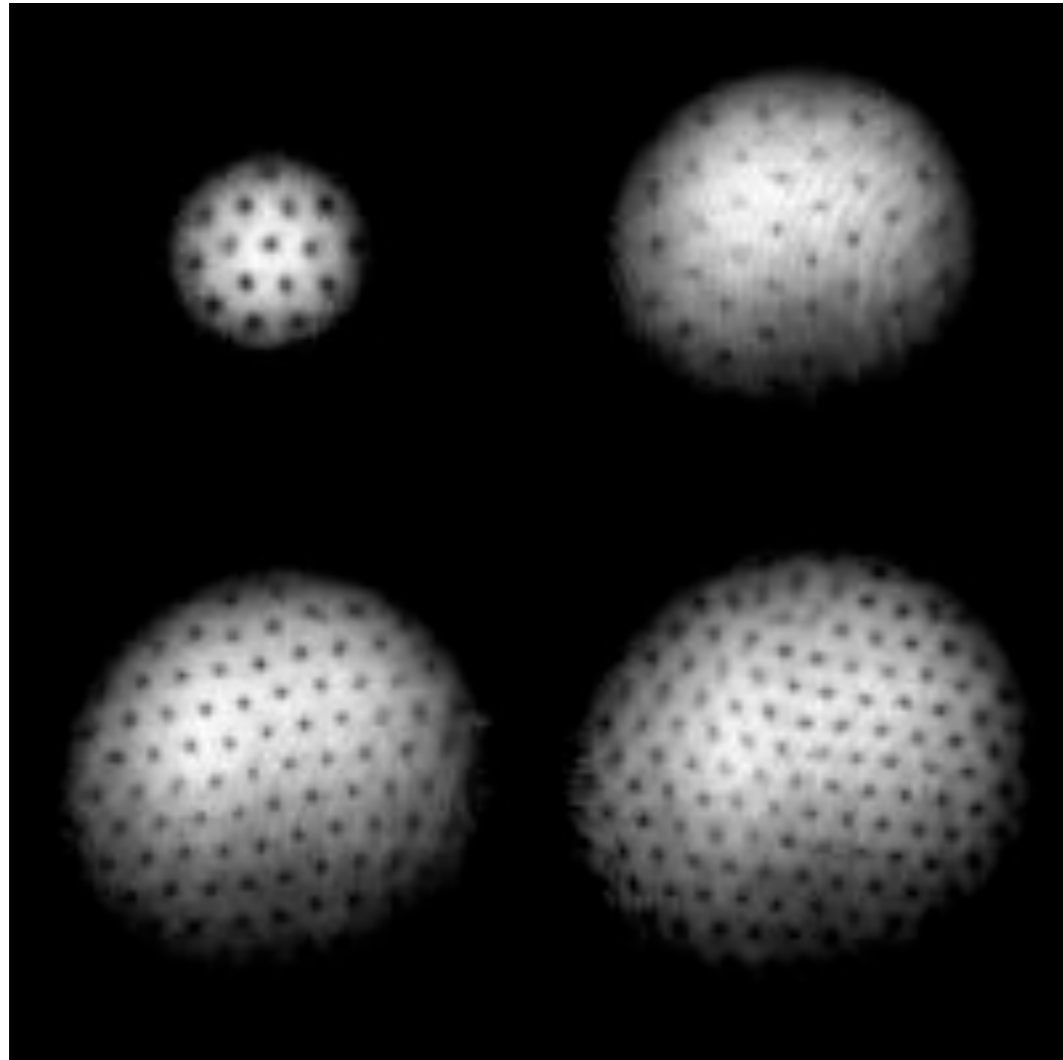
8

12



# *Large Abrikosov Vortex Lattices*

---

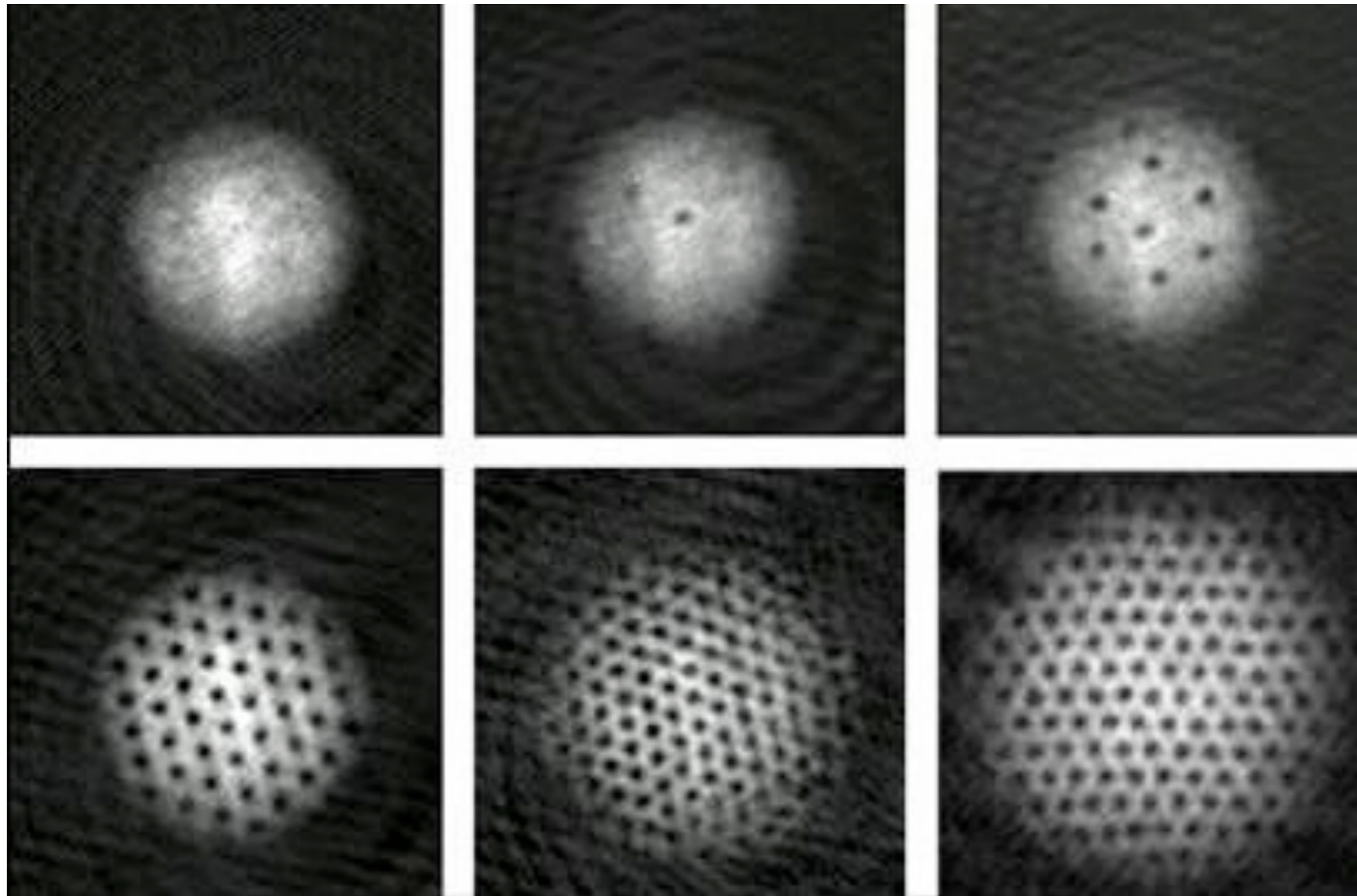


J. R. Abo-Shaeer et al. (2001)

Up to 150 vortices !

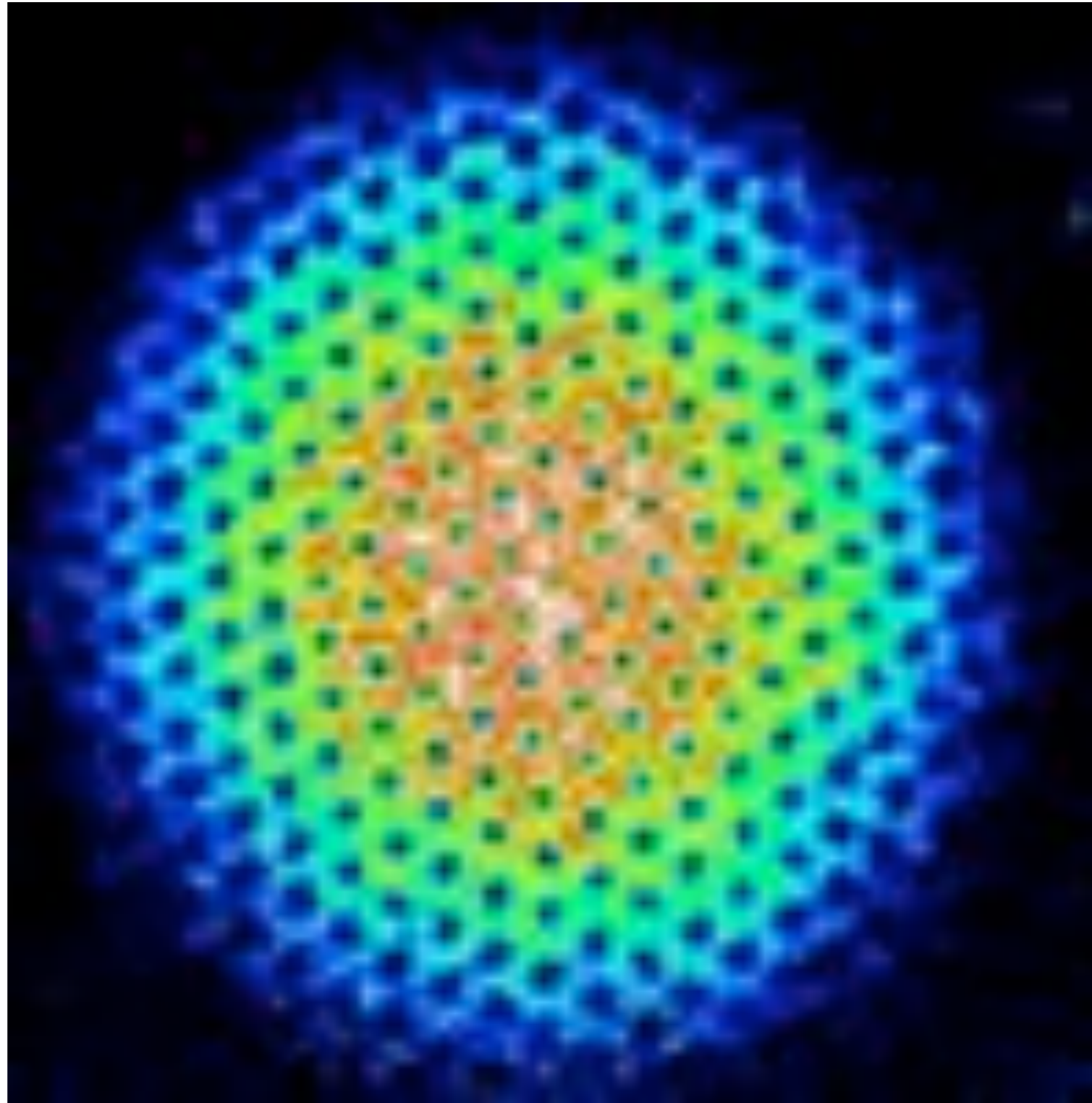
# *More Vortices*

---



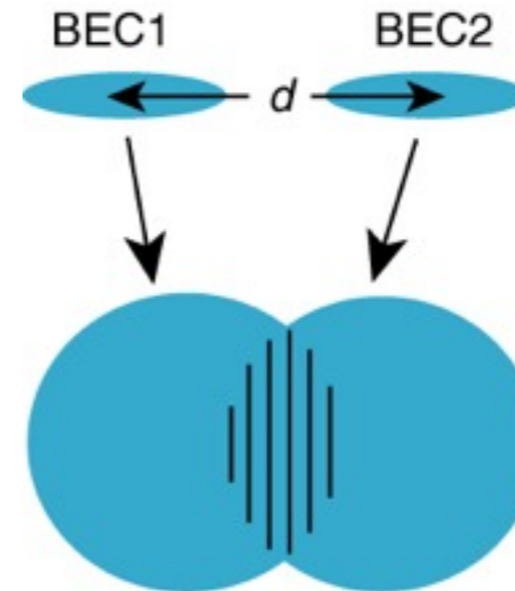
# *Even More...*

---



# Interference between two Bose-Einstein Condensates

Trapped  
BECs

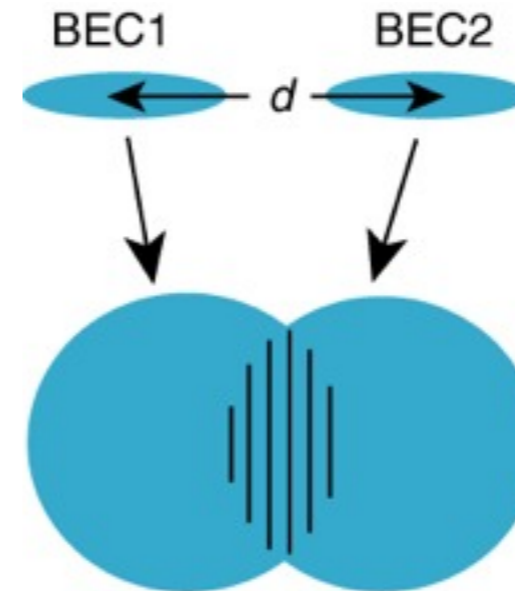


BECs after  
expansion time  $t$

$$\lambda = \frac{h}{m\Delta v} = \frac{ht}{md}$$

# Interference between two Bose-Einstein Condensates

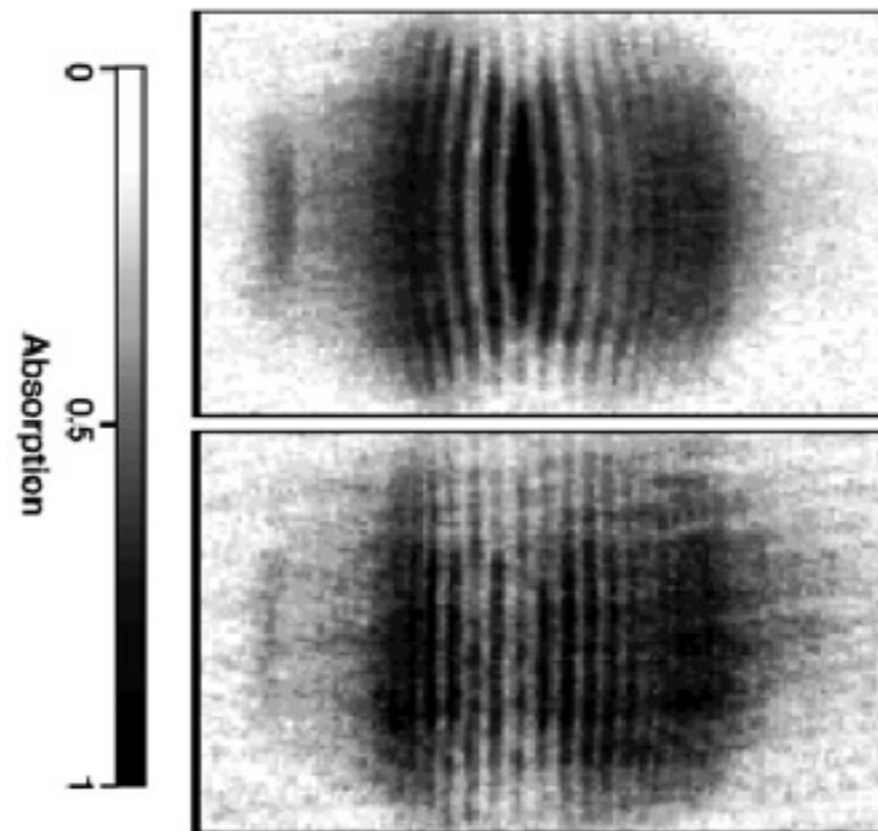
Trapped  
BECs



BECs after  
expansion time t

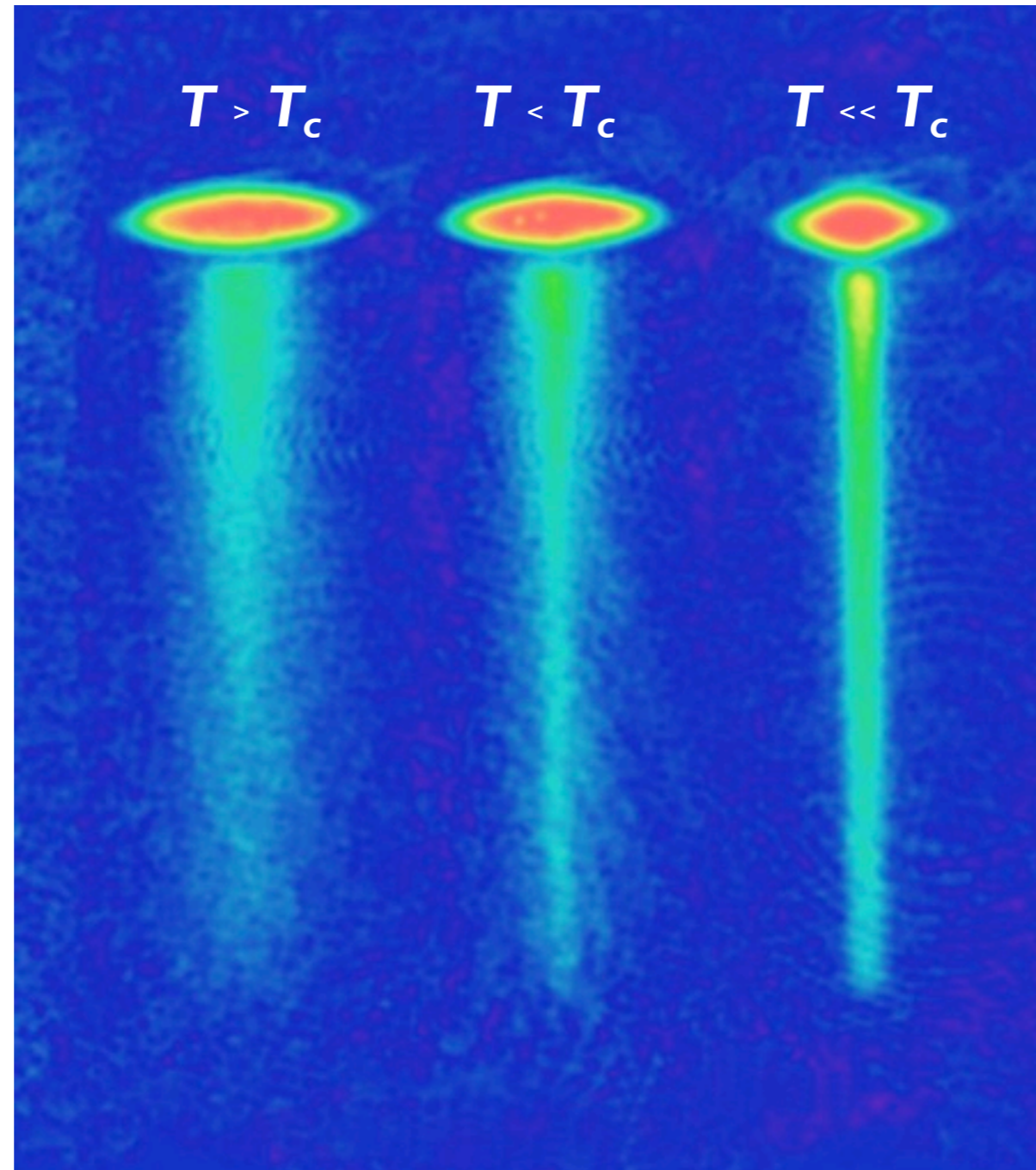
$$\lambda = \frac{h}{m\Delta v} = \frac{ht}{md}$$

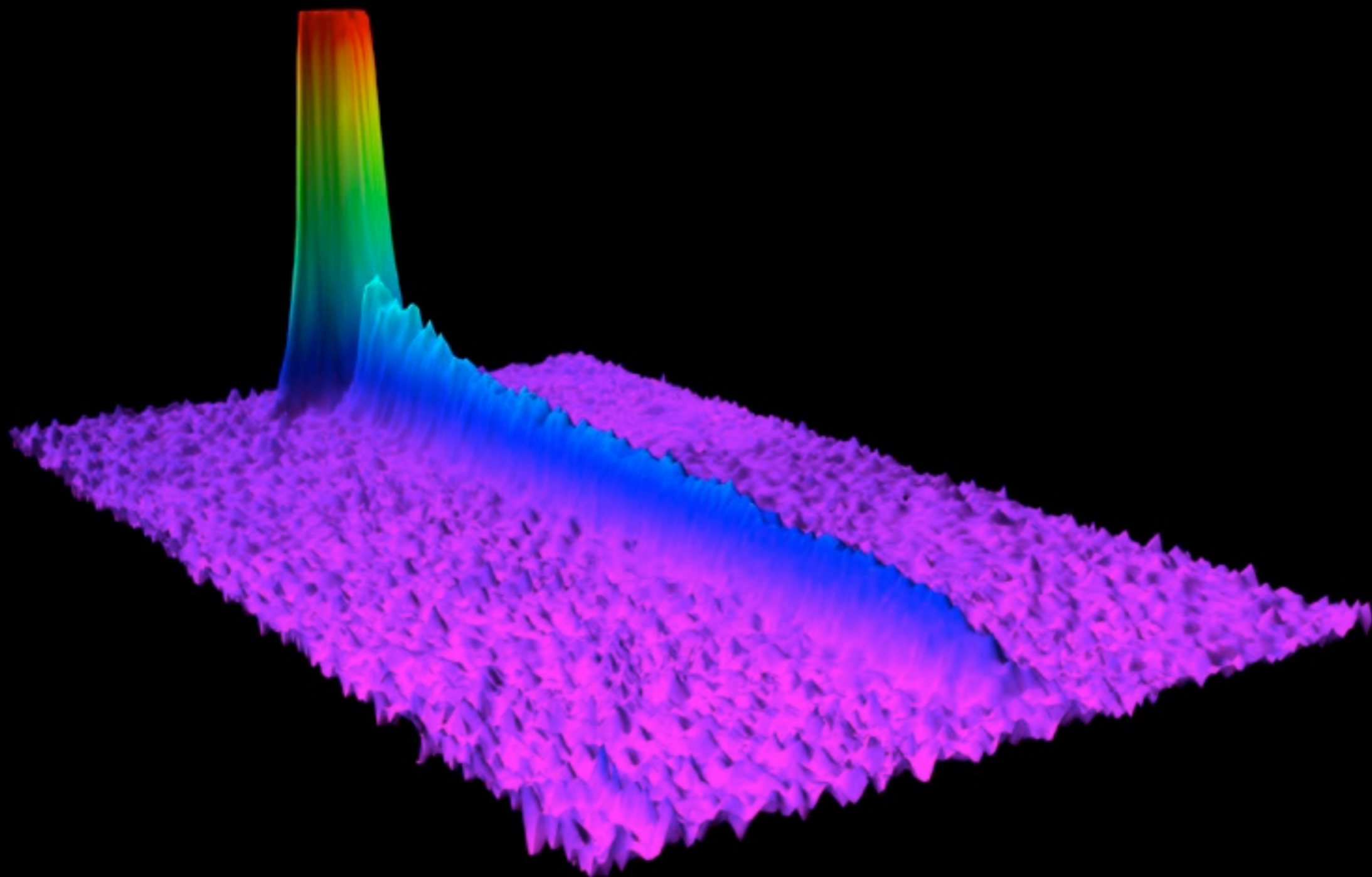
M. R. Andrews *et. al.*  
Science 275, ff. 637, 1997



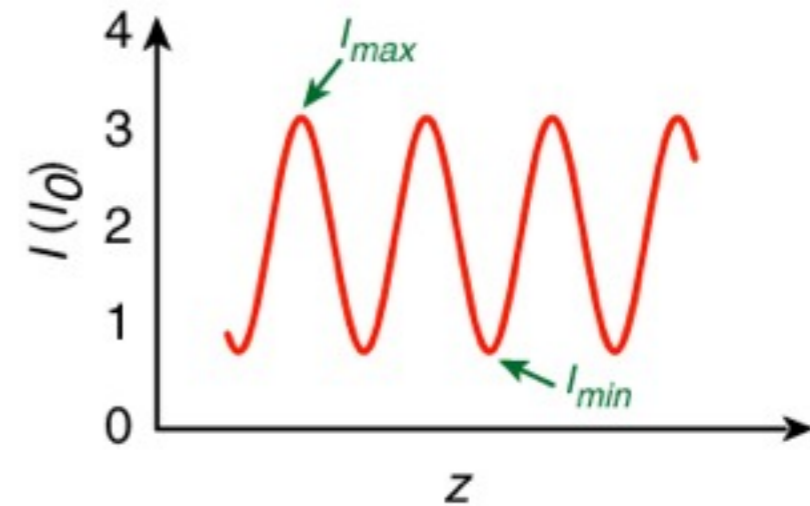
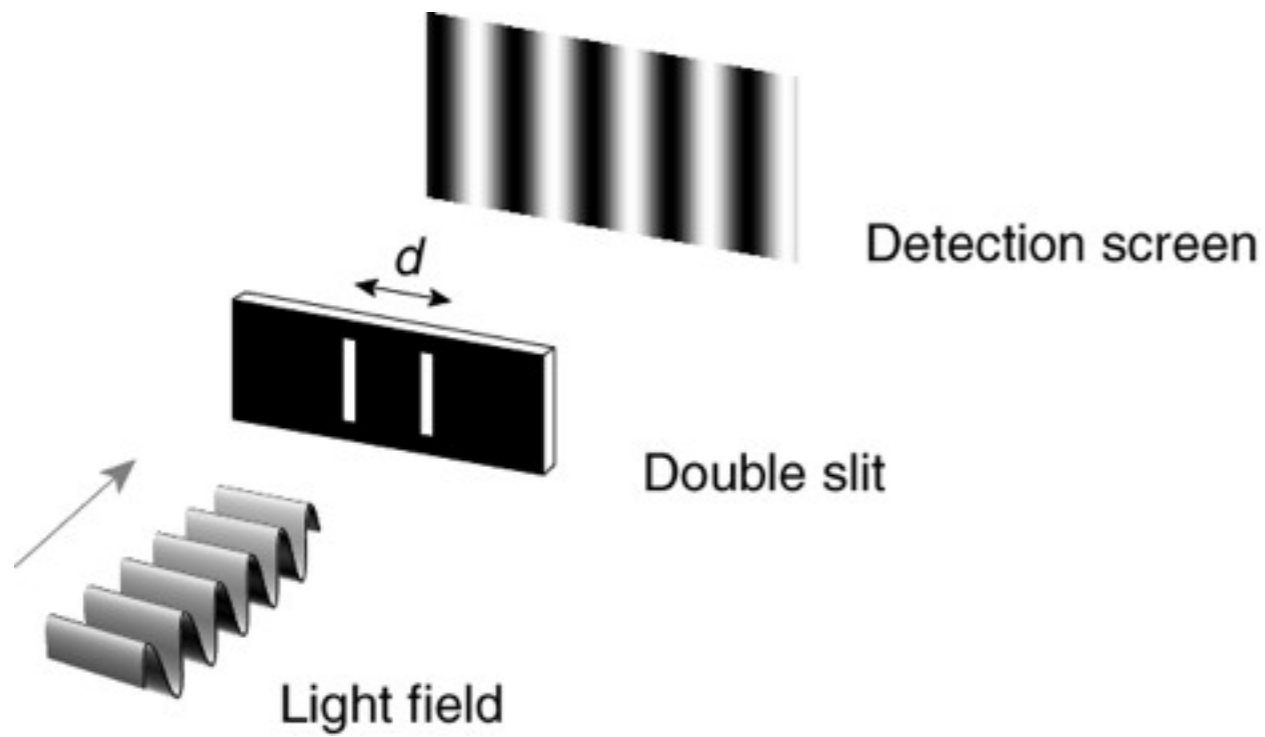
# Atomlaser am Phasenübergang

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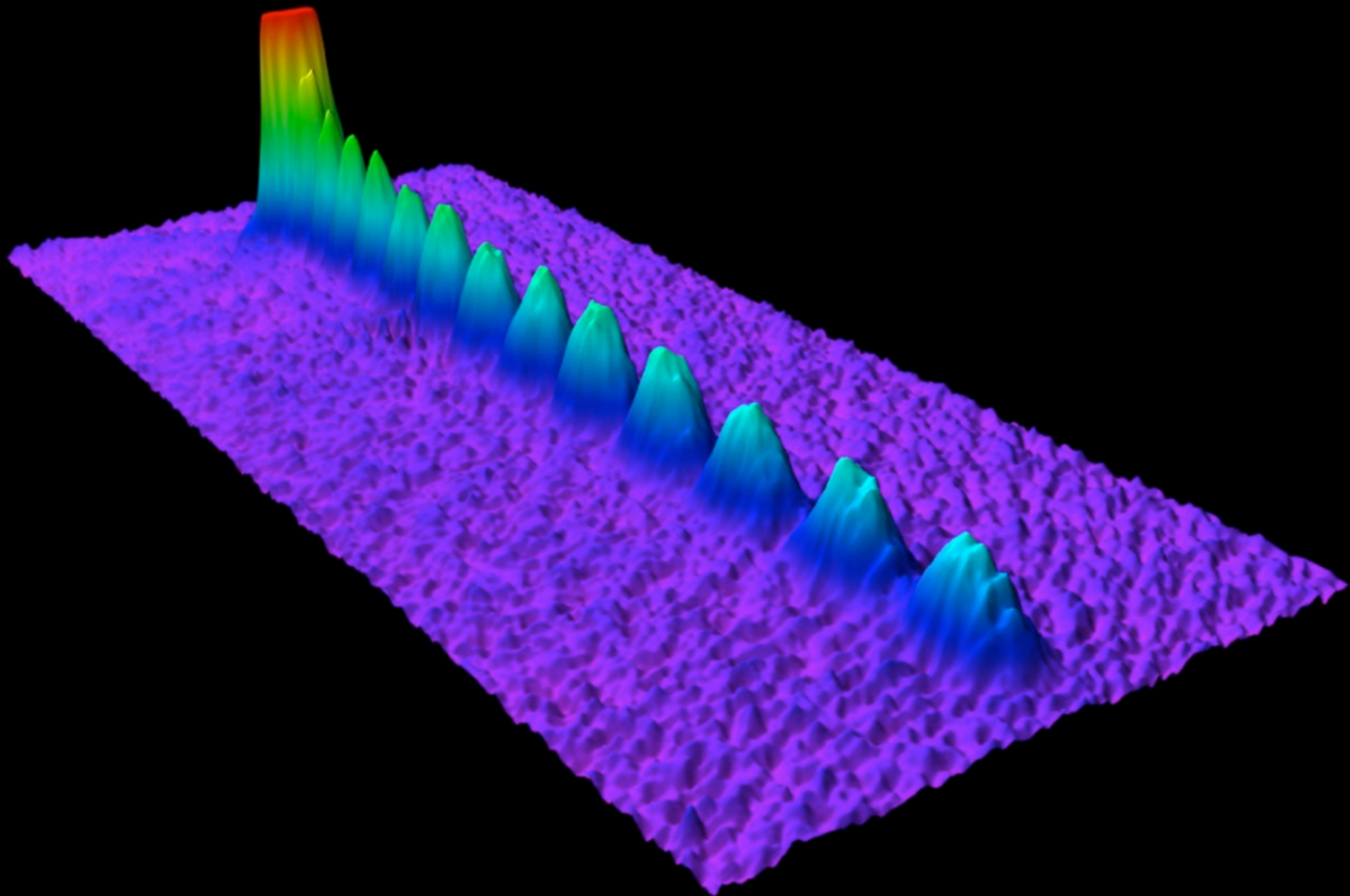




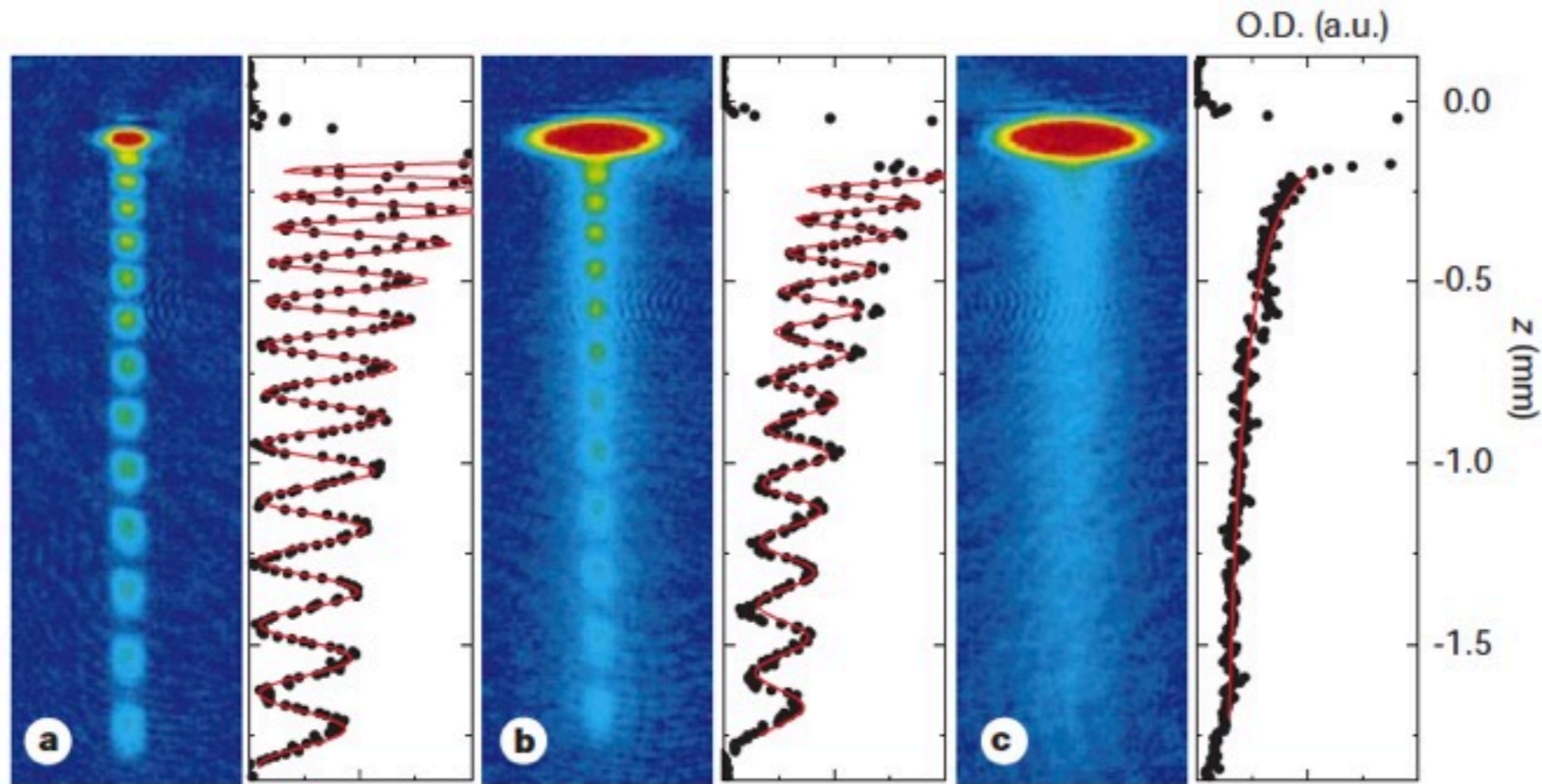
# Doppelspaltexperiment mit zwei Atomlasern



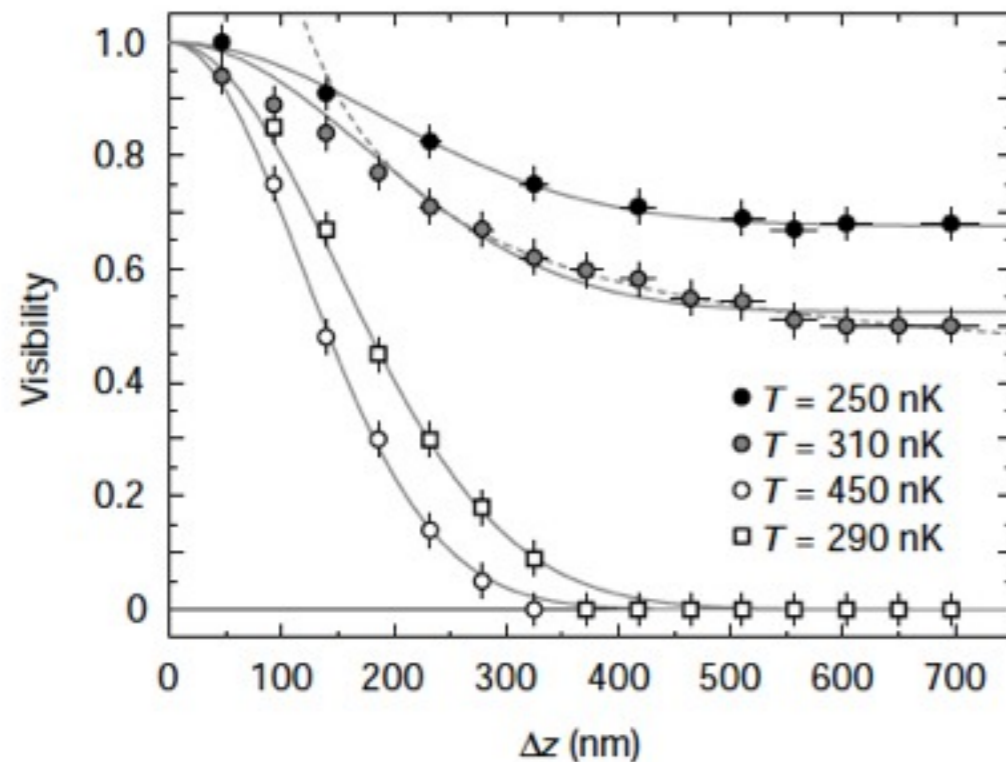
$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = g^{(1)}(\vec{r}_1, \vec{r}_2, \tau)$$



# Measuring First Order Coherence



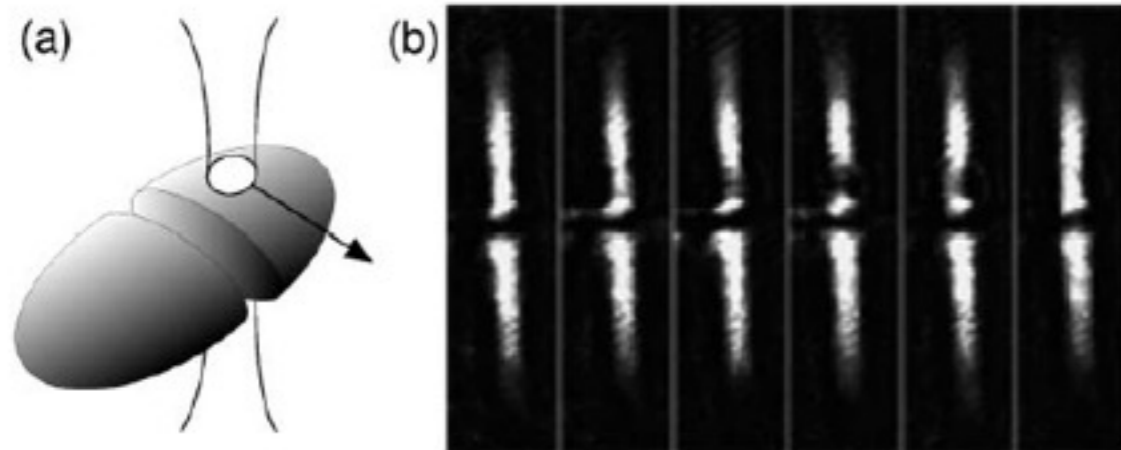
Interference pattern for fixed slit separation and variable temperature.



$$g^{(1)}(\Delta z) = \frac{\langle \hat{\psi}^+(0) \hat{\psi}(\Delta z) \rangle}{\sqrt{\langle \hat{\psi}^+(0) \hat{\psi}(0) \rangle} \sqrt{\langle \hat{\psi}^+(\Delta z) \hat{\psi}(\Delta z) \rangle}}$$

Condensate Shows Long Range Order (in 3D)

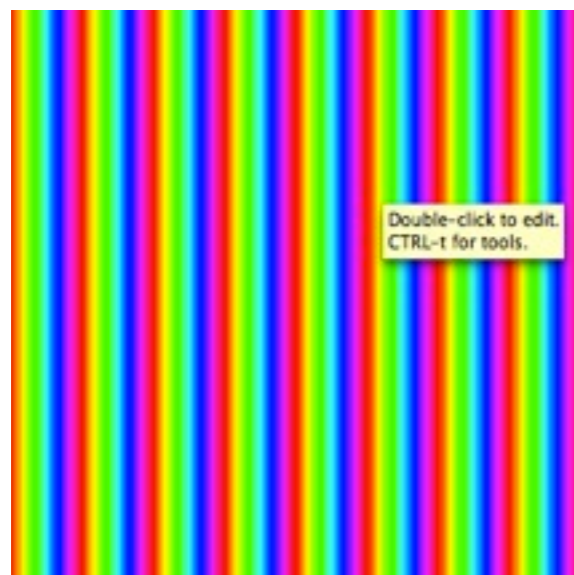
# Detecting the Phase Singularity of a Vortex



Make two condensates, one with a vortex and one without a vortex.

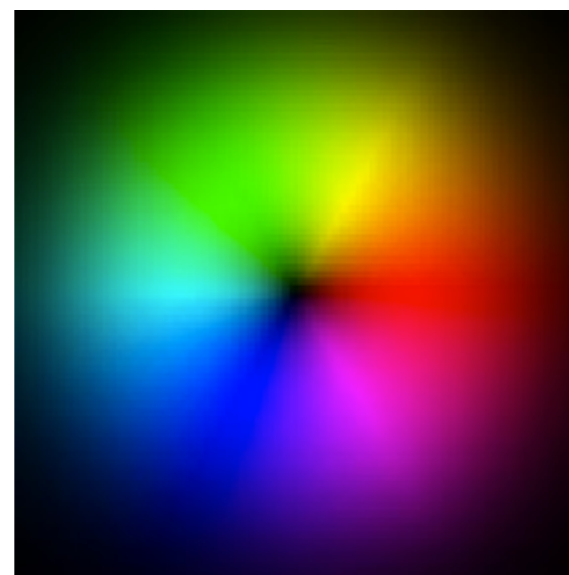
Let them interfere!

Expect Fork Dislocation in interference pattern, due to phase singularity!

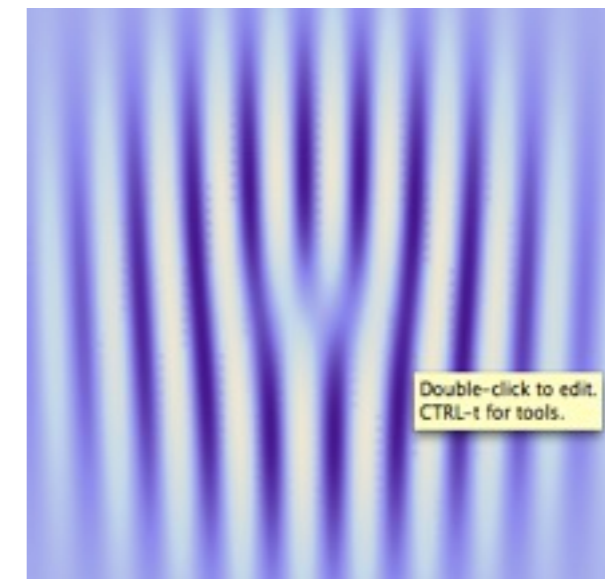
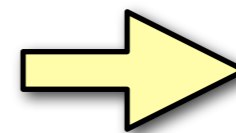


Plane wave  
wavefunction

+

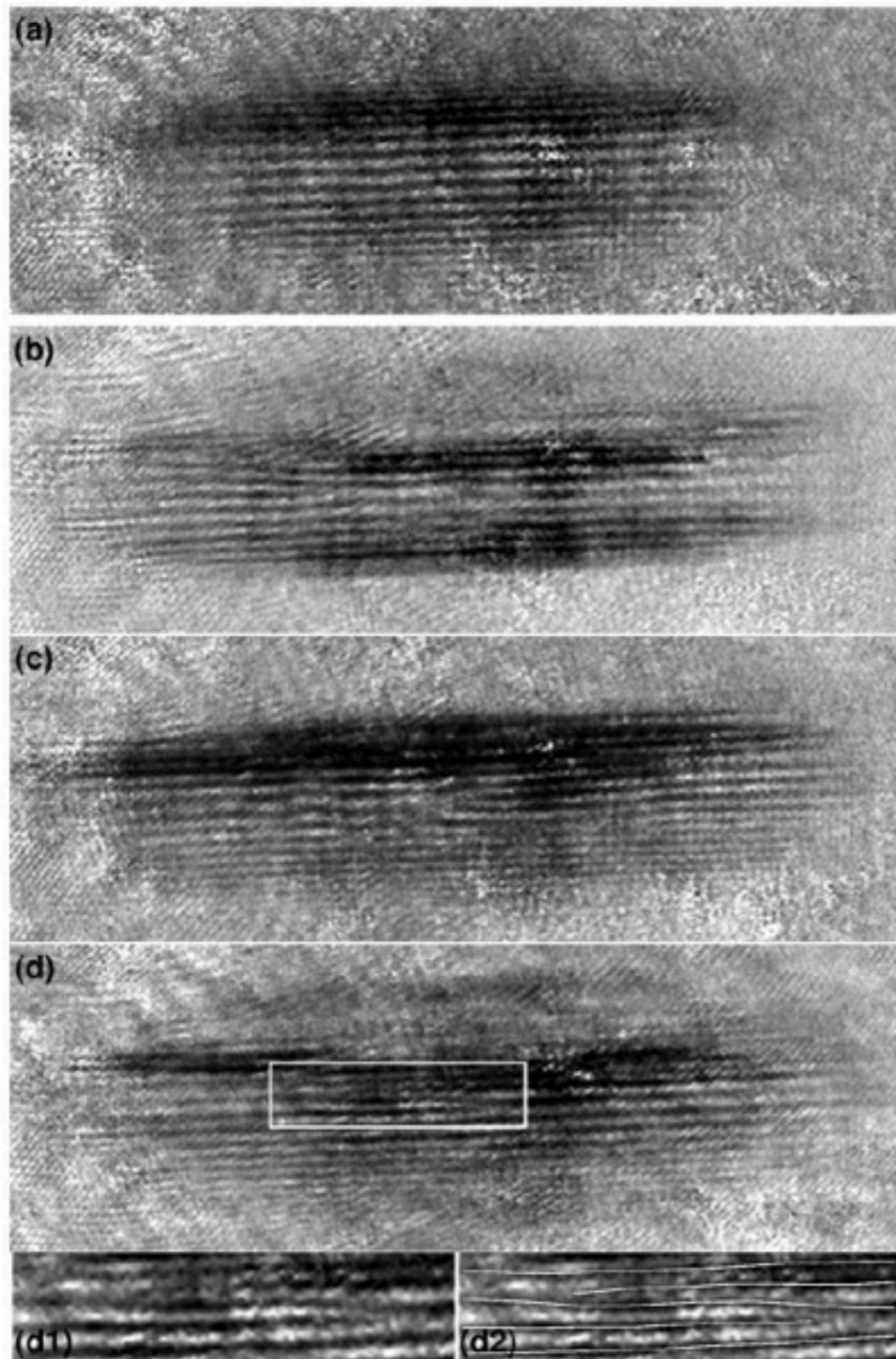


Vortex  
wavefunction



Interference  
pattern

# Observing the Phase Singularity



from S. Inouye et al.: PRL 87, 080402 (2001)  
see also: F. Chevy PRA 64, 031601 (2001)